Side Effects of the Lottery Panacea

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Abstract

Lotteries have long been used, wittingly or unwittingly, as a method for addressing the free rider problem in public goods. Morgan (2000) wrote the seminal paper demonstrating how a self-funding lottery can raise funds for a public good. His result is necessarily a second-best solution as the prize is funded out of ticket sales. Morgan and Sefton (2000) show that the theoretical predictions are supported by behavioural reality. Their work, indeed much of the literature on public goods, models public goods as a single generic public good which is demonstrably false. Moir (*forthcoming*) addresses this issue in a model with two public goods and shows both theoretically and behaviourally that a lottery can raise funds for the wrong public good. This current paper demonstrates the conditions under which a fund raising manager finds it impossible to design a lottery which blockades the entry of another lottery, instituted by a competing fund raising manager, for a different public good. In other words, we derive the conditions under which multiple lotteries exist in an environment where a social planner would otherwise prefer to see only one lottery.

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Side Effects of the Lottery Panacea

I. Introduction

Public good provision has always been a challenge for policy-makers. The tendency for individuals to free ride upon others' voluntary contributions means that a laissez-faire policy towards public goods will lead to under-provision (Samuelson 1954). The traditional solution to this dilemma has a government tax its citizenry and in return provide public goods. For a variety of reasons, governments have embraced "fiscal responsibility" through decreased program spending and active tax cuts. Nevertheless, governments and public/social programs need revenues in order to operate.

Both governments unwilling to tax and charities without the power to tax have turned to gambling, and more specifically lotteries, as a mechanism for revenue generation. While using lotteries to provide public goods seems to be a recent phenomenon, its history can be traced back to 1552 in the Republic of Venice (Seville 1999). In 1569 an English lottery was conducted to raise funds for the "'reparations of the havens and the strength of the realme and towards such other public good works'" (p. 20).

Morgan (2000) wrote the seminal paper on the use of self-funding lotteries as a fund raising tool for public good provision.[1] Instead of voluntarily contributing to a public good, agents purchase tickets to win some fixed prize. Ticket revenues in excess of the fixed prize are

[1] While Morgan uses the phrase "public goods" in his title, his model contains only a single generic public good. It was this issue that began the current line of research beginning with Moir (*forthcoming*). While Morgan's thesis implicitly suggests that a state-run lottery can generate general revenues from which the government can provide for the public good, explicit examples in the paper and Morgan and Sefton (2000) following, suggest that such lotteries could be used for fund raising by independent charitable organizations.

used to fund the public good.[2] Morgan shows that such self-funding lotteries work only when the public good is socially desirable (i.e., when the *social* return from the public good exceeds the private return from the private good). Most importantly, the results require only risk neutral agents; normally profit-making lotteries rely on the existence of risk-loving agents or agents with an asymmetric value function defined over changes in wealth (Tversky and Kahneman 1981). Morgan and Sefton (2000) conduct an experiment using a simple payoff (induced utility) function, linear in both the private and public good, and show that the theoretical results in Morgan hold behaviourally.[3]

As Moir (*forthcoming*; hereafter, this paper is simply referred to as Moir) demonstrates, the welfare-improving results expressed in Morgan do not necessarily follow when there are multiple public goods. A lottery can be used to fund the less desirable of two otherwise socially desirable public goods. While the lottery is still predicted to increase welfare as compared to the no-lottery free riding equilibrium, welfare would be further increased had only the most socially desirable public good received lottery support Moreover, behavioural results in Moir suggest that the inclusion of a lottery for the less desirable of two public goods actually decreases social

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[2] Implicit in Morgan's work is the assumption that the prize will in fact be awarded and that the net revenues from ticket sales will in fact be used to fund the public good for which the fund raising took place. We continue with this assumption.

[3] Moir (2005) argues that the presentation in Morgan and Sefton disguises the fact that behavioural support for the theory is qualitative. While public good provision increases with a lottery, the magnitude of the increase is quite small as the lottery crowds-out voluntary contributions made in the absence of a lottery. Absent a lottery, theory predicts zero contributions which has been shown to be behaviourally false.

payoff.[4] In the absence of a lottery, there are significant voluntary contributions to the more socially desirable public good. These voluntary contributions are crowded-out when a lottery supports just the less socially desirable public good.

The findings in Moir wave a red warning flag against the unmitigated use of lotteries as a method of improving social welfare. Multiple public goods significantly complicate the issue when compared to Morgan. Nevertheless, the results in Moir rely on the exogenous creation of a lottery in support of the less socially desirable public good. What if there were competing fund raising managers (hereafter referred to managers for convenience) who cared only for the net revenues raised for "their" public good? Cannot the manager for the more socially desirable public good crowd-out or blockade entry by the manager for the less socially desirable public good? Would any lottery for a less socially desirable public good disappear in the long run because of insufficient funding? If this was true, then the issues raised in Moir are interesting but inconsequential. We show that it is possible, in fact likely, for multiple charitable lotteries to simultaneously exist. The potential problems identified in Moir are likely to persist in the long run.

Bilodeau and Slivinski (1997) raise a similar issue with multiple not-for-profit firms vying to provide two public goods. Equilibrium outcomes include (a) a single firm providing multiple public goods, (b) a dominant firm providing multiple public goods with a fringe of specialty firms specializing in the production of a particular public good, or (c) a number of competing firms providing a mix of public goods. They propose that a single firm

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[4] Social welfare and social payoff need not necessarily coincide especially when a social welfare function accounts for income distribution via some weighting system. Here we use unweighted aggregate utility as a measure of social welfare.

providing multiple public goods is pareto superior. Our results, simultaneously extend Morgan and address cases (b) and (c) described by Bilodeau and Slivinski.

The lottery in Morgan works because those who buy a ticket but fail to win still receive a benefit from public good provision. This effect is large enough that such lotteries can be self-funding in that they raise enough for the prize and still provide for the public good. Here we show that charities that compete for funds through rival lotteries lead to an offsetting welfare loss. The prize they set not only has to attract ticket purchases and fund the prize, they must also compete against each other. In the more realistic world of cases (2) and (3) in Bilodeau and Slivinski, we show that competing lotteries further reduce welfare and argue that governments should not unquestioningly embrace Morgan's justification for lotteries as a fund-raising tool for the provision of public goods.

The remainder of the paper is structured as follows. In section II we use a linear model with homogeneous agents, comparable to the one used in Morgan and Sefton, to motivate our discussion. In section III we relax the homogeneity assumption to derive more general results and section IV concludes with policy prescriptions and an outline for further research.

II. Model

Both Morgan, and Bilodeau and Slivinksi, use quasi-linear utility functions to obtain their results. We instead opt for a linear utility function. Such a utility function is obviously an extreme and unrealistic assumption; absent of any mechanism, it predicts a Nash equilibrium with complete free riding (i.e., zero voluntary contributions) and total wealth contribution for social optimality. Quasi-linearity on the other hand, predicts an interior Nash and social optimum in the absence of a lottery *when wealth is not a binding constraint*. The loss in generality from linear utility is traded for gains in analytic tractability and exposition. If the problems we identify exist in this extreme situation then they are likely to be problematic in a more realistic environment. Moreover, given the "lottery panacea" that exists today, we feel it is important to raise this issue quickly. That said, a linear utility function may not be too extreme an assumption if people set a "charities" budget and use that to determine either contributions or charitable lottery expenditures[5] or if utility is quasi-linear and separable in private and public good consumption and the number of people is large relative to the marginal utility of the public good.[6]

Suppose there exist n agents each with the following utility function which is consistent with his/her preference ranking:

(1) $$u_i = x_i + m_i G + k_i H,$$

where x_i is an agent's private good consumption (normalized to have a return of 1), $G=\Sigma g_i$ is the aggregate level of public good G provided, and $H=\Sigma h_i$ is the aggregate level of public good H provided. The terms $m_i>0$and $k_i>0$ represent the marginal per capita return (MPCR) from public

[5] Fixed wealth limits the usefulness of an increase in the prize level supporting any particular lottery. Eventually people will spend no more money on a lottery despite an increase in the prize level. While this seems to be a problem with a linear utility function, it is also a problem for quasi-linear utility. Morgan shows that a self-funding lottery can provide a public good "arbitrarily close" to the socially optimal level with the appropriate prize. Unfortunately, arbitrary closeness requires a prize of infinite size which in turn requires infinite wealth if such a lottery is to be supported. Wealth is not infinite so the wealth constraint must be binding.

[6] In Childs and Moir (2005) we explore the impact of a paid fund-raiser who determines the appropriate prize level in an environment with quasi-linear utility and a single public good. Positive voluntary contributions are predicted at the Nash equilibrium with no lottery. However, individual equilibrium contributions tend towards zero when the number of individuals is large relative to the marginal individual return from public good provision. This situation can be approximated by a linear utility function.

good G and H respectively. Each agent faces a budget constraint,

(2) $$w_i = x_i + g_i + h_i,$$

where w_i represents the endowment of agent i.

To simplify the exposition, suppose agents are identical in endowments and preferences so that w_i=w, m_i=m, and k_i=k for all i. Also assume that m>k. This problem is interesting when 0<k<m<1, but nm>nk>1. While an individual's marginal investment in either public good provides a lower return to them than if the same investment had been made to the private good, by generating a return of m (or k) to all n individuals, the social return is higher for a marginal investment in either public good. Furthermore, while both public goods G and H are socially desirable (nm>1 and nk>1 respectively), public good G is *more* socially desirable.

If we assume that agents are solely interested in maximizing their own utility and give no consideration to others[7] then we can describe each agents' decision problem in the following manner: invest an additional unit of endowment in the good with the highest marginal per capita return. Agents' decisions can then be modelled as an integer programming problem.[8] We can calculate the marginal per capita return for investment in each good by successively partially differentiating **(1)** with respects to each choice variable. Thus we get:

(3) $$\partial u_i/\partial x_i = \text{MPCR}_x = 1,$$

(4) $$\partial u_i/\partial g_i = \text{MPCR}_G = m, \text{ and}$$

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[7] The standard zero conjectural variations Cournot-Nash behavioural assumption.

[8] Following Morgan, Moir models the decision-making process as a continuous choice problem, not an integer programming problem. However, in the experimental portion of both Morgan and Sefton and also in Moir, parameters and prize values for the lotteries were selected to lead to integer contribution values as equilibrium predictions. Moreover, subjects were restricted to make integer contributions out of their wealth endowments.

(5) $\partial u_i/\partial h_i = \text{MPCR}_H = k.$

Given $0<k<m<1$ by construction, each self-interested agent invests her entire endowment in x, generating utility of $u_i^N=w$ for each individual, and aggregate utility $U^N=nw$. In our stylized economy, this complete free riding Nash equilibrium is the least efficient method of public good provision.

A planner with a goal of maximizing aggregate utility would have each agent invest his entire endowment in G (the more socially desirable public good). Each agent receives utility $u_i^S=nmw$, resulting in an aggregate utility of $U^S=n^2mw$. Given $nm>1$ by construction, aggregate utility is greater at the social optimum as compared to the Nash equilibrium. Voluntary contribution of 100% of endowment is the first-best solution in this stylized environment, but is neither an equilibrium nor an expected outcome (see Davis and Holt (1993), Ledyard (1995), and Zelmer (2003) for surveys of public goods experiments).

Now suppose we allow for a self-funding lottery for either public good G, or H, or both. By self-funding, we mean that if wagers in a particular lottery do not meet or exceed the prize then the lottery is cancelled and wagers are refunded and invested in the private good. An expected utility function can then be described by:

(6) $\text{E}(u_i) = x_i + (g_i/G)P_G + (h_i/H)P_H + m(G-P_G) + k(H-P_H),$

where P_G and P_H represent the fixed prize for each of the public goods. Assuming self-interested agents maximize **(6)** subject to the budget constraint **(2)**, we calculate the appropriate (expected) MPCRs:

(7) $\partial \text{E}(u_i)/\partial x_i = \text{MPCR}_x = 1,$

(8) $\partial \text{E}(u_i)/\partial g_i = \text{MPCR}_G = (G_{-i}/G^2)P_G + m$, and

(9) $\partial E(u_i)/\partial h_i = \text{MPCR}_H = (H_{-i}/H^2)P_H + k$,

where G_{-i} (H_{-i}) represents the aggregate wagers on the lottery for public good G (H) made by others (i.e., $G_{-i}=G-g_i$ and $H_{-i}=H-h_i$).

For public good G (H), the MPCR is decreasing in g_i (h_i), decreasing in G_{-i} (H_{-i}) for all values of $G_{-i}>g_i$ ($H_{-i}>h_i$) and increasing otherwise[9], increasing in P_G (P_H), and increasing in m (k). Suppose we are in an equilibrium with some prize, $P_G>0$, for a lottery for public good G and $P_H=0$. The manager for H can institute a small-prize lottery, $P_H>0$, which raises MPCR_H and can lead to wagers in the lottery for public good H. Herein lies the complication for competing managers and is a problem we return to below.

In this environment, we define the second-best solution as a single lottery placed on the most socially desirable public good and achieving the maximum possible aggregate utility after funding the prize (i.e., $P_G>0$, and $P_H=0$). Recalling that agents are identical, and assuming like agents act alike, then in equilibrium, $G^*=ng_i^*$ and $G_{-i}^*=(n-1)g_i^*$. We can rewrite **(8)** and **(9)** as:

(8') $\text{MPCR}_G = [(n-1)/n^2](1/g_i^*)P_G + m$, and

(9') $\text{MPCR}_H = k$.

As $k<1$ by construction, then **(9')** implies $h_i^*=0$ for all i. Equating **(8')** to **(7)** we solve for the individual equilibrium wager for the lottery for public good G,

(10) $g_i^* = [(n-1)/(n^2(1-m))]P_G$, and

(11) $G^* = ng_i^* = [(n-1)/(n-nm)]P_G$.

We present **(11)** in Figure 1. Note that $\partial G^*/\partial P_G = [(n-1)/(n-nm)]$ is greater than 1 when

[9] Partially differentiating **(8)** with respects to G_{-i}, $\Omega = \partial\text{MPCR}_G/\partial G_{-i} = (P_G g_i - P_G G_{-i})/G^3$. Assuming positive values for G, $\Omega<0$ for $G_{-i}>g_i$, and $\Omega>0$ for $G_{-i}<g_i$. Likewise for H_{-i}.

$nm>1$. The self-funding criterion is met when the public good is socially desirable.[10] Moreover, net provision of G (i.e., G-P_G) grows with the size of the prize up to the wealth constraint (see the solid and dashed lines in Figure 1).

The minimum prize necessary to extract all wealth as wagers for a lottery supporting G is calculated by setting $g_i^*=w$ in **(10)**,

(12) $$P_G^{\min} = w(1\text{-}m)[n^2/(n-1)],$$

which is identified in Figure 1. At $P_G^{\min}$, $g_i^*=w$ so $MPCR_G=1$. A prize up to, and including $P_G^{\min}$ is necessarily utility-improving as compared to the case of complete free riding with no lottery. A prize larger than $P_G^{\min}$ reduces aggregate utility because the prize is larger than necessary to extract all wealth in the form of ticket purchases. Thus, the second-best solution to this problem is to institute a lottery (perhaps managed by a government) with a prize of $P_G^{\min}$ for the most socially desirable public good. Of course, identifying the most socially desirable public good is extremely difficult if not impossible, and prohibitively costly. Instead, the government could, and does, allow charities to organize their own lotteries.

For the sake of argument, we define a competing manager as a manager for a charity/public good who cares only about the net revenue raised for her charity.[11] We then pose the question, "Can the manager for the more socially desirable public good set a prize that

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[10] If we allow lotteries to run which have a prize conditional upon ticket sales (e.g., 50/50 draws or any prize expressed as a fraction of ticket sales, or parimutuel lotteries consistent with most nationally- or state-run lotteries in the world) then the self-funding criterion of Morgan's model is no longer applicable so the internal check of social desirability disappears.

[11] This manager could be motivated by self-interest in the charity or could be a hired individual whose pay (or continuing employment) is based, in whole or in part, upon net revenues raised for her charity. See Childs and Moir (2005) for further discussion.

prevents the competing manager for the less socially desirable public good from instituting a self-funding lottery?" If the answer to this question is "No", then it is clear that in the absence of specific policy limiting lotteries to only those public goods which are most socially desirable, we will end up at best, at a third-best solution with multiple public goods competing for contributions through the use of lotteries.[12]

Let $P_G^{max}=nw$ be the largest possible prize in our stylized economy (identified in Figure 1). Because $P_G^{max}>P_G^{min}$ by **(12)**, we know that $g_i^*=w$ for all i. In other words, a prize is offered which extracts all wealth which is then returned lump-sum to a single individual for private consumption. Note that this leads to a aggregate utility solution equivalent to the original case of complete free riding in the absence of all lotteries, with the exception that the utility distribution is entirely skewed to one individual. Further define $P_G^{\sim}$ such that $P_G^{min}\leq P_G^{\sim}\leq P_G^{max}$. Aggregate utility at $P_G^{\sim}$ is less than aggregate utility at P_G^{min} because the wealth constraint is binding. The higher prize induces no new ticket sales and drains resources away from providing more of the public good.

Now suppose agents sequentially purchase a lottery ticket at a cost of one unit of endowment. The last person choosing where to invest her last unit of endowment must rank the expected MPCRs for the two lotteries and the certain return from private good investment. We set the prize equal to $P_G^{\sim}$ to establish our comparison criteria and examine the results at the two extreme values identified above. In this case, **(8)** reduces to,

[12] Competing lotteries provide less aggregate utility when compared to a single lottery supporting only the most socially desirable public good, which in turn provides less aggregate utility than the social planner's solution. At worst, competing lotteries can provide aggregate utility equal to the aggregate utility at the Nash equilibrium absent a lottery.

(13) $\text{MPCR}_{G^{\sim}} = [((n\text{-}1)w)/(nw\text{-}1)^2]P_{G^{\sim}} + m,$

where G_{-i}=$(n$-$1)w$ and G=nw-1 reflects the single unit of endowment the last person has yet to place in the lottery for G. Now consider the competing manager for public good H. She need only set a prize that causes one person to wager one unit of endowment in the lottery for public good H. If the prize necessary to motivate this move is less than one, then the manager has met her goal and generated positive net revenues for her public good. In the absence of a prize for public good H, we know that h_i=0 for all i. Define H_{-i}/H^2 at H_{-i}=H=0 as 1 (infinite probability equals certainty), then **(9)** reduces to,

(14) $\text{MPCR}_H = P_H + k.$

The manager for public good H can profitably induce one person to switch a wager from the lottery for public good G to the lottery for public good H if she selects $0<P_H^*<1$ such that $\text{MPCR}_{G^{\sim}}<\text{MPCR}_H$. These conditions can be combined and expressed as,

(15) $[((n\text{-}1)w)/(nw\text{-}1)^2]P_{G^{\sim}} + m - k < P_H^* < 1.$

Let $\phi = [((n\text{-}1)w)/(nw\text{-}1)^2]P_{G^{\sim}}$.

First consider $P_{G^{\sim}}$=$P_G^{\min}$. Then $\phi = [(nw)^2/(nw\text{-}1)^2](1\text{-}m)$. The first term in the square parentheses is a value greater than 1, but approaching 1 from above as nw gets arbitrarily large.[13] Thus, as nw gets arbitrarily large, ϕ=(1-m), and **(15)** reduces to,

(15a) $1 - k < P_H^* < 1,$

which is always solvable for P_H^* because $0<k<1$ by construction.

Result 1: *An optimally designed lottery which provides the largest net revenue for the most*

[13] For instance, using the parameters from Moir – n=3 and w=20 – then the term in the square parentheses is approximately 1.0342.

socially desirable public good can never blockade entry into the lottery market by a competing manager raising funds for the less socially desirable public good.

Therefore all possible candidates for blockading prizes must necessarily be greater than the optimal single prize and this implies an aggregate utility reduction as net revenues for public good G fall for $P_G^{\sim} > P_G^{\min}$.

Now consider $P_G^{\sim} = P_G^{\max}$. Then $\phi = [((n\text{-}1)w)/(nw\text{-}1)^2]nw$, and for all positive but finite values of n and w, $\phi<1$. We can rewrite **(15)** as,

(15b) $$[((n-1)w)/(nw-1)^2]nw + m - k < P_H^* < 1.$$

For a wide variety of parameters of n, w, m, and k, there exists P_H^* satisfying the conditions in **(15b)** – a lottery run by a competing manager for the less socially desirable public good can successfully enter the lottery market. We have used $P_G^{\max}=nw>P_G^{\min}$ as a breakpoint in our analysis.[14] In this highly stylized economy of extremes, if a lottery which extracts all wealth and returns it as a prize to a single individual cannot blockade entry into a market, then no prize can.

Result 2: *Under a wide variety of parameters, it is possible and indeed likely, that no lottery*

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[14] If we follow the implementation of the lottery in Morgan and later in Moir, then when the lottery fails to be self-funding, ticket purchases are refunded and it is assumed (enforced) that these refunds are invested in the private good. Suppose $P_G=nw$ and no other lottery exists. The aggregate utility in this instance is equal to the Nash equilibrium level of welfare with the important exception that the entire wealth will be concentrated in one person's (the winner's) hands. Now further suppose that P_H^* exists and is implemented by a competing manager. As argued, the last person would purchase a single ticket in the lottery for public good H. Now, the net revenue for public good G is -1; the lottery fails the self-funding criterion and everyone has their ticket purchases refunded. This means n-1 agents each receive a return of w for their private good consumption. The last person receives w-1 return from his private good consumption. At the same time, he generates $(1\text{-}P_H^*)k$ for each of the n individuals including himself, and receives P_H^* back as a prize (as the sole entrant in the lottery). This is necessarily an improvement over the no-lottery Nash equilibrium and over a lottery which blockades competition by setting the prize equal to aggregate wealth. This situation is untenable, as the manager for public good G can lower P_G and create a self-funding lottery once again (see *Result 4* below).

prize for the more socially desirable public good exists which can blockade entry of new lottery by a competitive manager from a less socially desirable public good.

Suppose, contrary to construction, that the two public goods are equally socially desirable (m=k) and $P_G^{\sim}$=P_G^{max}=nw. Now **(15)** reduces to,

(15c) $\phi < P_H^* < 1.$

which is always true as ϕ<1. It is never possible with competing managers to prevent a lottery from developing for an equally socially desirable good.

Result 3: *When two public goods are equally socially desirable, there exists no prize in a lottery for one socially desirable public good which would blockade entry of a lottery for the other socially desirable good.*

Result 4: *By extension of Result 3, as two public goods approach equal social desirability (i.e., as k→m) it becomes increasingly difficult to design a prize for the lottery supporting the more socially desirable public good to prevent entry of a lottery for the less socially desirable public good.*

For nm arbitrarily large (but finite), nw-1$\approx$$nw$ and **(15)** reduces to,

(15d) $[1-(1/n)] + m - k < P_H^* < 1,$

which also represents the solution to the problem if we ignore the integer constraints imposed upon behaviour. Furthermore, the limit of ϕ as $n \rightarrow \infty$ is,

(15e) $1 + m - k < P_H^* < 1,$

which has no solution for P_H^* unless m<k, which is false by construction. A corollary to **(15e)** is that a lottery supporting a less socially desirable public good can never blockade entry by a competing manager for a lottery supporting a more socially desirable good.

Result 5: *A lottery supporting a less socially desirable public good can never blockade a competing manager from entering the lottery market to fund a more socially desirable public good.*

Using parameters from the experiment in Moir, n=3, m=0.75, and k=0.50, then **(15b)** equals 0.939457, so if $P_H^* = 0.9395$ the last person would wager her last unit of endowment on the smaller prize for the less socially desirable public good.[15] In this instance, indeed under a wide variety of parameters, we arrive at a third-best solution, with multiple lotteries. This is necessarily welfare reducing as it diverts funds from the more socially desirable public good to the less socially desirable one. Moreover, if competing managers are paid out of net revenues, aggregate utility is reduced even further.

III. Relaxing the Homogeneity Assumption

Suppose we drop the homogeneity assumption so agent j faces possibly distinct values for w_j, m_j, and k_j.[16] We impose the following restrictions: **(a)** $\Sigma m_i > \Sigma k_i > 1$, and **(b)** $nm_j > 1$ and $nk_j > 1$ for all j. Restriction **(a)** is analogous to our assumption that public good G is more socially desirable and public good H less so. A social-planner in a first-best solution would mandate 100% contribution of endowments to good G. In a second-best solution, they would like to see a lottery exist solely for good G. Restriction **(b)** ensures that each agent, projecting his/her

[15] Indeed, it is quite possible that a higher value of P_H can be sustained as a net revenue generating lottery – numerical analysis suggests that this is the case. What we show here is that it is very difficult to block the entry of a competing lottery and the act of blockading reduces aggregate utility.

[16] It is important to note that the results here include no equilibrium analysis, but, as we motivate our results using an integer programming model of agent behaviour, this is not an issue. Exploration of equilibrium results depends upon distributional assumptions and are left to be explored in another paper.

preferences upon others, believes both good G and good H to be socially desirable. It need not be the case that $m_j > k_j$ – individuals need not show the same preference towards each public good.

Under these conditions, we can re-write **(15b)** as,

(16) $$[(\Sigma w_i - w_j)/(\Sigma w_i - 1)^2]\Sigma w_i + m_j - k_j < P_H^* < 1,$$

where i indexes all agents. For large but finite values of Σw_i, **(16)** approaches,

(16a) $$[1 - (w_j/\Sigma w_i)] + m_j - k_j < P_H^* < 1.$$

It is conceivably easier to find solutions for P_H^* in **(16a)** than for the various versions of equation **(15)**. Consider only heterogeneity in wealth. A relatively rich person in a large economy (i.e., w_j is large relative to Σw_i) drives the terms in the squared parentheses down, making it easier to find a candidate value for P_H^*. With our hypothetical model of sequential unit investments, the richest person will be the last person to make an investment decision. All previous investments in the good G lottery would have driven the MPCR_G down, thus increasing the likelihood for a value for P_H^* to exist and a lottery for good H to arise.

Now consider only heterogeneity in preferences. For some agents $k_j > m_j$ – they prefer good H to good G – which according to *Result 5* necessarily implies that a candidate value for P_H^* exists. These two entirely plausible but not nearly exhaustive degrees of heterogeneity suggest that it is even more likely to see multiple lotteries arise.

IV. Conclusions and Policy Prescription

The lottery method of fund raising translates the free rider problem of positive externalities into a common pool resource problem with managers competing for a limited source of funds. In the lottery-free case, the Cournot-Nash equilibrium predicts zero contributions to any public good where utility is linear in all goods. The advent of a single lottery in the single public good case or for the most socially desirable public good in the multiple public good case, leads to a second-best outcome. This second-best outcome involves high levels of funding for the most socially desirable good, with limited losses to the lottery prize. In the case of multiple public goods, unrestricted entry into the lottery market leads to a third-best solution under a wide variety of parameters. Aggregate utility reduction occurs not only because funds are diverted from the more socially desirable public good to the less socially desirable public good, but also because prizes enter into direct competition with each other (i.e., they are higher than necessary and provide no additional public good of either sort). In the absence of policy restricting entry into the lottery market, there will likely be a proliferation of lotteries. This is observed in the field, with increasing numbers of games of chance in support of a wide array of public goods. If all public goods are not created equal this leads to a third-best outcome.

A solution to this problem is to implement policy that limits entry into the lottery market. In the Canadian context this responsibility lies both with the federal and provincial governments. There are a variety of different policy measures that can be taken to limit this problem. The Ontario Early Years Foundation for instance requires that groups accepting funds from the foundation refrain from other fund-raising activities. Alternatively, a single centrally-run lottery can be instituted and surplus funds distributed across public goods.[17] The government can mandate fixed license fees for lotteries, or can require that compensation schemes for hired managers must include a fixed component in addition to a proportion based upon revenues

[17] This is a variant upon the Bilodeau and Slivinski result.

raised. These fixed fees must then be paid out of net revenues.[18]

We have shown that potential welfare losses exist when lotteries are used as fund raising methods for public goods even in the absence of transactions costs. Obvious sources of transactions costs include the use of paid managers, the costs of ticket printing and lottery promotion, and the costs of cancelling any lotteries not meeting the self-funding criterion.[19] The absence of transactions costs makes the outcomes discussed in this paper the least damaging possible. The inclusion of transactions costs will make the deviation from the second-best solution even greater as multiple managers must be paid.

There are ample opportunities to extend this work. First, we should explore the same issues with more diverse utility structures including non-linear utility functions and heterogeneous agents. The effects of revenue-conditional prizes (e.g., 50-50 draws and parimutuel lotteries) need to be studied in a multiple public good framework. A revenue-conditional prize obviates the self-funding criterion and no longer guarantees that only socially desirable public goods are funded. Revenue-conditional prizes can also increase the likelihood of competitive lotteries entering the lottery market. If we accept the argument that lotteries should be centrally managed and funds for various public goods distributed by a central authority, then voting mechanisms need to be explored which identify a social ranking of public goods.

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[18] This amounts to setting a minimum net revenue constraint thus making **(15)** harder to satisfy. This constraint can be implemented even when managers come from a pool of volunteers.

[19] Of course, a lottery could be run at a loss, but this too imposes an additional social cost.

Despite the simple stylistic design of this economic model, our study provides an important warning; while the use of lotteries as a fund raising tool for a public good can increase aggregate utility, the unmitigated use of lotteries in a multiple public good environment can have important negative utility effects. Competition for a fixed pool of resources (i.e., donations, ticket sales, auction dollars), leads to additional inefficiencies when applied in a fund raising context. Policy-makers need to pay attention to this issue.

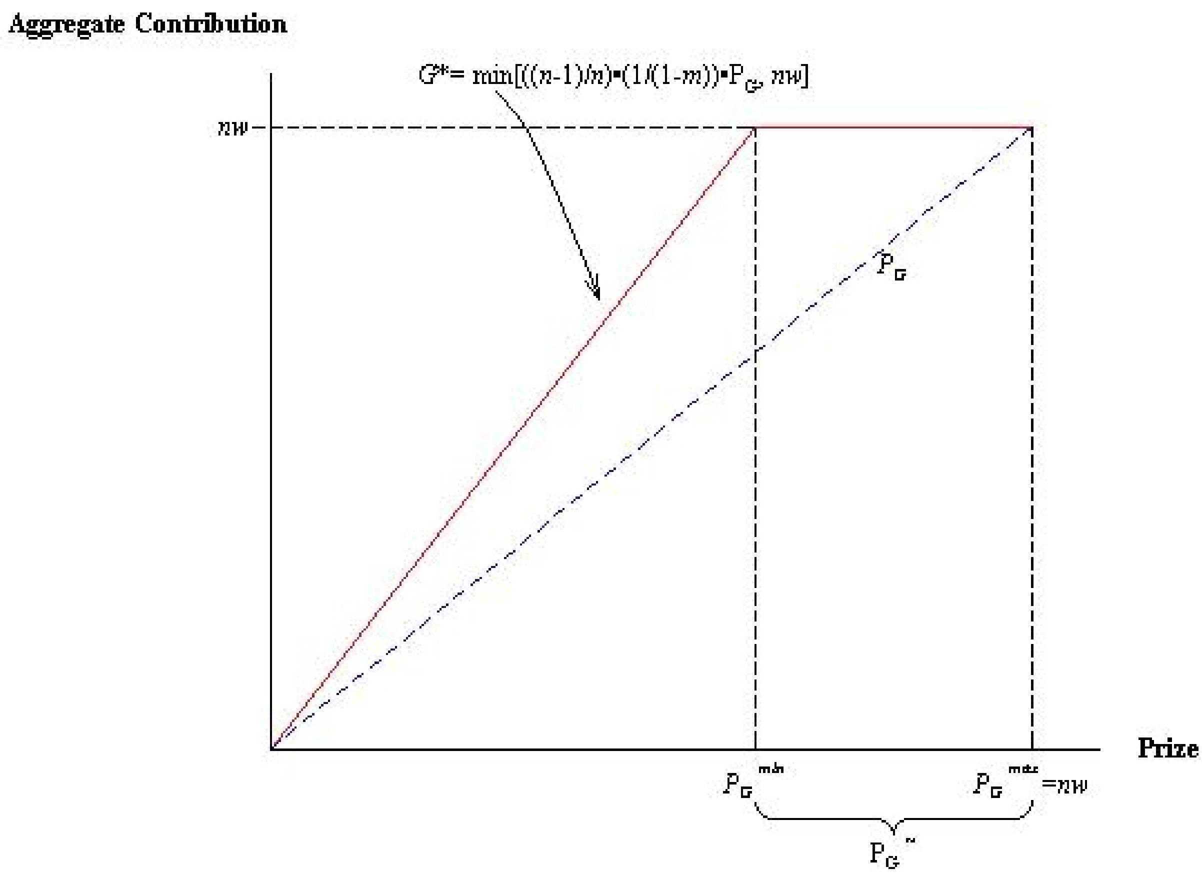

Figure 1: Aggregate Contributions and Prize Levels

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