A COMPUTABLE GENERAL EQUILIBRIUM SIMULATION OF THE U.S.-CHINA TRADE WAR

by

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Abstract

On April 3rd 2018, the United States of America announced 25% tariffs on goods imported from China, which applied to approximately $46 billion (USD) of trade, and China retaliated by placing tariffs of equal value on imports coming from the United States. As the two biggest economies in the world, China and the U.S. comprise more than 40% of global trade, and their international trade policies have a significant influence on the global economy.

This study examines the possible consequence of the U.S.-China trade war at the aggregate level by applying an Armington type of static computable general equilibrium model. Simulation results show that both the U.S. and China experience a welfare loss, and the magnitude of loss in China is larger than that of the U.S.. If China retaliates, the U.S. is expected to gain in the manufacturing sector, and China may suffer from a more significant loss in welfare.

Chapter 1

Introduction

The 45th U.S. President Donald Trump, during his election campaign in 2016, regarded the loss of millions of domestic jobs, especially in the manufacturing sector as due to the huge trade deficit with China. As one of the U.S.'s biggest trading partners, according to the WTO Statistics Database, in 2017 China and the United States comprise more than 40% of global trade. Their trade relations have a major influence on the Canadian and other economies, and a trade spat is expected to create a impact shock to the global economy.

Trade tensions escalated in March 2018, when the U.S. imposed multiple rounds of anti-dumping tariffs and launched a 'Section 301' signed by Trump investigating China's malicious behavior of stealing intellectual property, and threatened extra tariffs on the Chinese imports. Trump fulfilled his promise that had worried the world, and in April 2018, the United States announced a 25% tariff applying to $46 billion (USD) of 1,333 Chinese imported goods, followed by a tit-for-tat response by China of the exact same value, which signaled the beginning of the U.S.-China trade spat.

In May 2018, the Chinese Vice Prime Minister, He Liu, came to the United States in order to negotiate the future trade relationship with the hope of reducing tariffs. Despite his effort, the United States announced another round of $50 billion (USD) worth of tariffs on 1,000 products from China, and China again retaliated by the same amount. The trade dispute was then escalated further. Up to now, the U.S. has already imposed $250 billion (USD) of tariffs on Chinese imports. On the other hand, China imposed $110 billion (USD) on U.S. imports. The two country's leaders, Donald Trump and Xi Jinping, discussed the trade relationship at the 2019 G20 summit in Osaka, but the trade war might still be far from an end.

Based on some of general equilibrium models used in previous international trade papers, this thesis provides a simulation analysis to point out which country and which industries would be the relative winners after the escalation of tariffs by observing the relative change in welfare, output, and the terms of trade. The simulation contributes to the literature by explaining the general equilibrium models, mathematical functions, the Social Accounting Matrix database construction, calibration and computational procedures.

This paper uses 2014 as the base data year, examining the economy at the aggregate level based on the World Input-Output File 2014 and the Socio Economic Accounts Release 2016 database [35]. Here is a brief summarization of the simulation results:

Simulation results show that both the U.S. and China experience a welfare loss, and the magnitude of loss in China is larger than that of the U.S.. If China retaliates, the U.S. is expected to gain in production and employment in the manufacturing sector, and China may suffer from a more significant loss in welfare than the U.S.. The escalation of tariff war enlarges both countries' welfare loss, however, it benefits the U.S. manufacturing sector if China retaliates with the same percentage of tariff. Elasticity parameters are key determinants in the simulation process which describe the responsiveness of quantity with respect to the change in price, and its value may significantly affect the result. My results find that the outcome of the U.S.-China trade war is not sensitive with respect to the elasticity of technology transformation between capital and labor in the range 0.7 to 1.25. For an elasticity of substitution between manufactured goods and non-manufactured goods between from 0.5 and 2, higher value reduces the loss in welfare but hurts the U.S.'s manufacturing production and employment as well as the volume of global trade.

The presentation of the thesis is as follows. Chapter 2 provides a review of trade policy analysis by using gravity models and general equilibrium models, then talks about simulations in computable general equilibrium (CGE) models, particularly simulations about the ongoing China-U.S. trade war. Chapter 3 provides a static CGE prototype (Nobuhiro, 2004)[16] for this thesis's simulation with a brief summary of each sector. The last subsection of Chapter 3 provides each sector's optimal solution. Chapter 4 presents data collection, the Social Accounting Matrix (SAM) construction, and calibration, followed by the results of an elasticity sensitivity test on production and consumption functions in Chapter 5. Chapter 6, the last chapter, summarizes the findings and suggests possible improvements in the simulation process used in this paper.

Chapter 2

Literature Review

2.1 Trade Simulation by Gravity and General Equilibrium

Gravity models and computable general equilibrium models are the two most common tools for trade policy quantitative analysis. The former focuses on trade policy through empirical analysis, whereas the latter predicts the economy in the future by treating the status today as a general equilibrium outcome. For international trade, CGE compares the economic status before and after the implementation of trade policies such as changes in tariffs or quotas.

Gravity models are applied to conduct empirical quantitative analysis and are widely used to observe the impact of policy on the flows. Tinbergen(1962)[36] and Poyhonen(1963)[32] were the first economists applying Newton's law of universal gravitation to examine the influence of bilateral trade flows. In a gravity model, the trade flow between counties is determined by the mass or the size of the two economies. The supply force is generated by the aggregate income of exporters, and the demand force is generated by the aggregate income of importers. The gravity distance is captured by transportation costs. The estimation of gravity models uses time series data, and regional, language, trade zone or barriers, and it can also empirically predicts the the impact of tariff on trade flows through the average change. But the limitation of a gravity model is that it only explains the pattern of trade rather than a direct estimation of change in welfare (Olena and Aaron, 2007)[17]

A Computable General Equilibrium (CGE) model is an instrument to analyze market economies. The model encompasses all major parts of economics activities, and it focuses on the interrelationships between each different agent. In this large-scale framework, the economy goes through a circular flow (Presented in Figure 2.1): producers produce and pay wages to labors and the rental fee to capital. Households get wage and the rental income then choose to consume and save, during which process, households and firms pay tax to the government who also consumes and saves. In contrast, partial equilibrium model only looks at a single component, but ignores a much larger effect on other sectors. CGE can examine the effect of an exogenous shock to a large scale economic model, such as the reform of environmental policies, trade policies and taxation.

The study of general equilibrium dates back to 1874, *Elements of Pure Economics*, in which the French economist Leon Walras attempted to apply neoclassical economic theory to model price and agents' motivation. Kenneth Arrow and Gerard Debreu

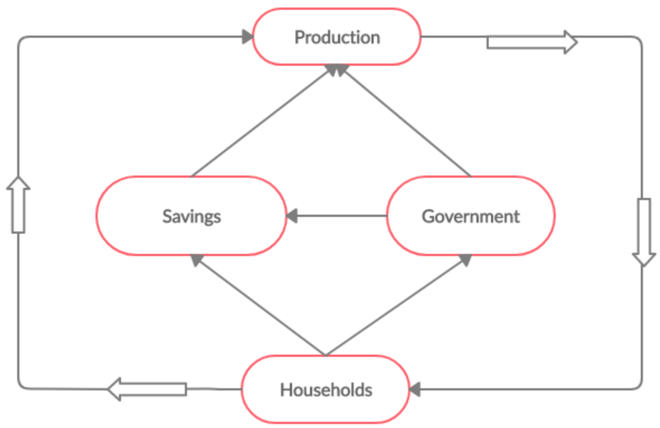

Figure 2.1: A National Economy Circular Flow

(1954)[2], Debreu (1959)[8], Debreu (1974)[9] contributed to prove the existence of general equilibrium in a competitive market with the same number of variables and equations as a sufficient condition. The difficulty of proving general equilibrium's existence was also bypassed through other methods by Von Neumann (1945)[38], then similar to Debreu, the work was proved by McKenzie (1959)[29]. Incorporating with the Walras' Law stating that the examined market must be in equilibrium if all other markets are in equilibrium, and the two-persons zero-sum game suggested Von Neumann (1928)[39], it applies that the trade surplus of each country should add up to zero in trade. Both general equilibrium principles play significant roles in the simulation in this paper.

Manlivaud (2012)[28] suggested that general equilibrium models seldom arrive at definite conclusion without precise quantitative analysis, however, the simulation result heavily relies on the model structure. From 1960s, much of the international trade policy literature have been based on Armington (1969)[1] with heterogeneous regional goods and the constant return to scale perfect competition assumption. The heterogeneity assumption tells that despite the same type of good, consumers have preference towards its country of origin, so that consumers would select between domestic goods or foreign goods, and the goods produced from different countries, the geographically differentiated goods. Unlike the classical Ricardo and Heckscher-Ohlin trade models that are based on comparative advantage theory and factor endowments, the Armington model helps to reflect real world trade patterns where countries export and import the same product category. When the relevant markets are all large, competitive market general equilibrium provides a sufficiently good approximation, and in the 1880s, the Irish economist Edgeworth showed that markets tend to be competitive if the number of participants increase indefinitely (Malinvaud, 2012)[28]. Based on the Armington structure, modern computable trade analysis combines this with the neoclassical general equilibrium supported by Hicks (1939)[15] to give a standard formalization of consumers and producers. In the neoclassical theory the demand side consists of utility maximizing households and the supply side are firms who choose the production level to maximize profits.

2.2 Static CGE Model Simulation

The standard static CGE model is a static "real" model which removes all the nominal concerns by assuming complete price flexibility and it solves for one period, focusing on the change in the long-run. Unlike the need for time series data in dynamic models or gravity models, static CGE models in contrast only require the data from a single period such as social accounting matrix (SAM) based on input-output table (I/O) where large scale simulation the interaction between the major components of the economy is reflected. SAM shows the transaction between institutions, so that the data are organized to provide a visual display of the transactions as circular flow of income and spending (Bufisher, 2017)[6]. Leontief (1936, 1951)[22][23] started to work on the input-output file to observe the interactions across industries caused by the policy change. Whereas an I/O table only presents how each industry relates to each other and their transactions in production. And the construction of a SAM needs I/O and other sources such as the employment data, tax and household spending rate. Plugging in the number from SAM to quantify the impact rather than purely predicting the directional change through economic theory also shows the "computable" aspect of CGE models.

In the mid-1980s, Tome Hertel from Purdue University started the Global Trade Analysis Project which played an important role in the Uruguay Round Negotiations, the formation of ASEAN and South Africa Economic Conference by applying the Armington type of CGE for simulation. Despite more advanced models with such features as product differentiation at the firm level and imperfect competition (Krugman, 1980)[21], (Melitz, 2003)[30], most of the conventional computable studies still adopt the Armington competitive market and constant return to scale assumptions to model trade policy reform.

2.3 Literature on U.S.-China Trade War

There have been some papers discussing the possible effect of China-U.S. trade war since 2012, some of which treat the economy at an aggregate level using the Armington assumption in general equilibrium with the nested utility assumption. They assume that the world market has two goods and three hierarchies of consumption. The goods are classified as tradable and non-tradable to aggregate the volume of trade, and the three levels of consumption include manufacturing and non-manufacturing, imported or domestic, and different countries of origin. The optimal consumption bundle is nested so that the optimal amount of consumption is determined only after getting the solution of the optimal choice of their best purchasing strategy for the total composite good. Then the import de-composites into the bottom regional level [1].

Papers applying the nested-utility Armington assumption such as (Dong and Whalley, 2012) based on the 5 countries, 2009 data and (Li et al., 2018)[25] on 2014 data, applied static CGE and nested utility functions to model the trade motivation, simulate that the U.S. gains on welfare if China takes no action, but will lose if China retaliates, while China loses of all indicators and China gets hurt more than the United States. The loss on both sides would be more severe if the tariff rate increases. The global economy will be hurt at the aggregate level due to the trade sanctions from the two largest economies. However, other countries will gain more access into the China and the U.S. markets mainly due to trade diversion. The second paper by Li et al. (2018)[25] also simulates the non-tariff war scenario which can be interpreted as policies such as anti-dumping sanctions that cannot be collected by the

[1] See Appendix A for nested-utility optimization procedure (Li et al., 2018)[25]

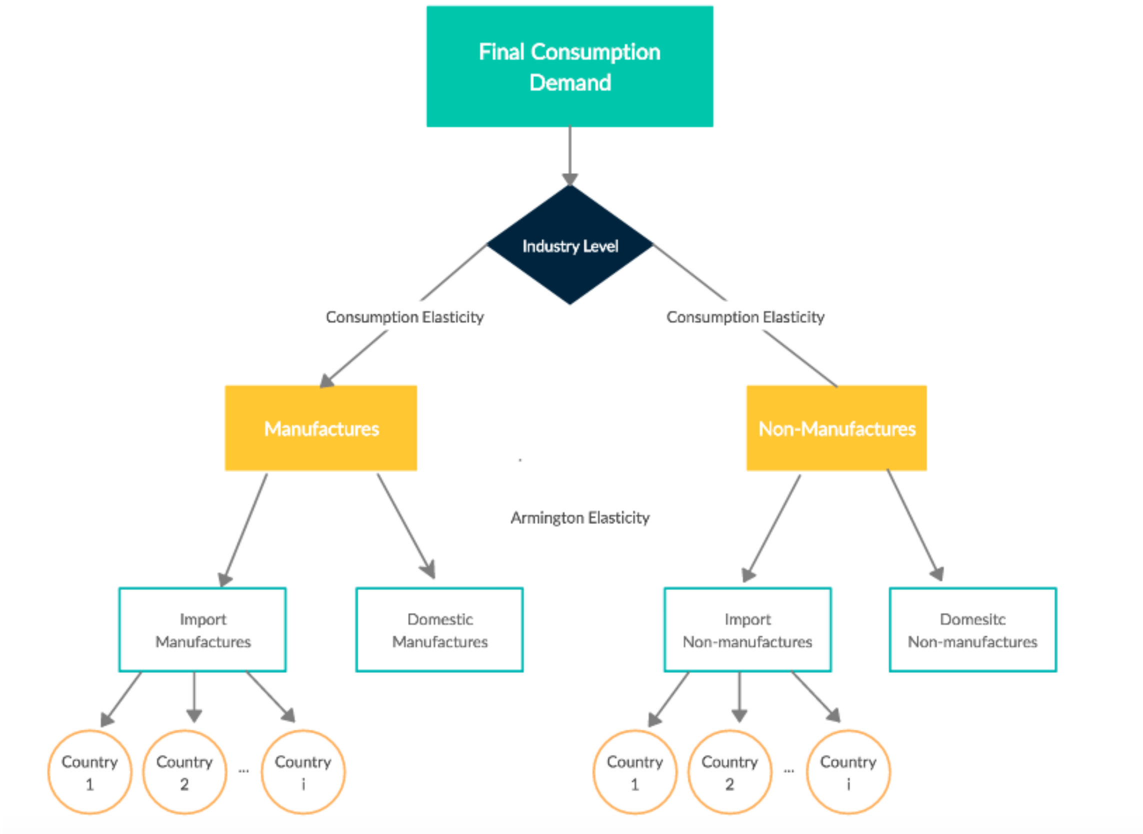

Figure 2.2: Nested Armington Assumption

government. Nevertheless, non-tariff war results in a similar consequence as the tariff war. Figure 2.2 presents the Armington type of CGE model with the nested-utility assumption

Simulation based on industry level such as the Global Trade Analysis Project (GTAP) and its database is another popular stream of CGE modeling started by Hertel (1997)[14]. Instead of the vague aggregated total trade volume, GTAP modelers examine the variation of each economic sector due to higher import tariff rate. Carvalho et al. (2019)[7]point out that from disaggregated level under HS6 codes classification listing 1363 goods, the steel and aluminum production in the U.S. is expected to increase, but the U.S. and China witness a reduction in welfare, as well as the rest of the world. Similarly, China suffers a comparatively smaller loss if it retaliates. The analysis of Rosyadi and Widodo (2018)[33] and Bollen and Romagosa (2018)[5] were based on the same data set under 140 countries and 57 industries. Tsutsumi (2018)[37] also used this original data and further aggregated to 16 countries and 12 industries based on GTAP. At the disaggregate level, different from the Armington assumption analysis, all of the above three papers argue that no matter whether if China retaliates or not, the United States would lose in welfare measured by equivalent income variation, but to a larger extent if China imposes tariffs on U.S. goods, and China will lose due to a higher tariff. Different conclusions regarding the relative change of the U.S. welfare results from different model structures, namely, if the trade is observed at the aggregate level or the disaggregate level.

In much of the Armington type of trade simulation papers, the elasticity of substitution σ is a key determinant in the model. As one of the most important features of CES utility function, elasticity parameters are usually plugged in exogenously from econometric analysis or from previous literature to take up one degree of freedom. It determines the responsiveness of consumers or producers due to the change in price or income. Also, it describes the preference between input factors or consumption goods: how hard it is to compensate one unit loss of a good while keeping the same utility. For example, high Armington elasticity indicates that more foreign goods are needed to compensate the loss of one unit of domestic good. In other words, the response due to price change is comparatively higher when the value of elasticity is larger. Different elasticity values may result in vastly different consequences, and the magnitude of the equation and model is directly linked with the value of the elasticity.

Previous literatures have discussed the elasticity values. Jomini et al. (1991)[18] proposed that the Armington substitution elasticity between domestic and imported goods is set equal to 2. This argument was further tested by Liu et al. (1998)[27], back-casted by Dimaranan (2002)[10] based on GTAP 5.0 database, and applied in GTAP 7.0 documentation (Hertel et al., 2014)[13]. Dong and Whalley (2012)[11] suggested the "rule of 2" principle for Armington elasticity in CGE simulation, and the paper also set the top level consumption substitution elasticity between manufactures and nonmanufactures goods as 0.5 in all regions. The other elasticity parameter, the production input transformation elasticity, is not consistent in the recent study. Karabarbounis and Neiman (2013)[19] proposed the elasticity value as 1.25, whereas, Muek et al. (2017)[31] measured it as 0.7.

By using the already-known elasticities parameters, share parameters and the scaling parameters are calculated by plugging in the real world data. This process is called calibration where modelers also solve the optimization with numerically defined functions and parameters, then should the endogenous result replicate the plugged-in data.

Some literatures on U.S. China trade war conducted sensitivity test, Li et al. (2018)[25] shows that a higher Armington elasticity increases both the U.S. and China GDP and welfare, but reduces the rest of world GDP and welfare. Whereas, a higher

Table 2.1: Relative Change of GDP or Welfare under U.S.-China Mutual Tariff War in the Simulation Literature

Paper	**Structure**	**U.S.**	**China**	**ROW**	**World**
Shagdar and Nakajima (2018)[34]	Industry Level	-	-	+	-
Carvalho, et al (2019)[7]	Industry Level	-	-	+	-
Bollen and Rojas-Romagosa (2018)[5]	Industry Level	-	-	+	-
Li et al. (2018)[26]	Industry Level	-	-	+	-
Lin et al. (2018)[25]	Aggregate	+	-	+/ -[2]	-
Li (2017)[24]	Aggregate	-	-	*	-
Dong and Whalley (2011)[11]	Aggregate	+	-	-	- (If more than 50%)[3]

"+" means the relative change is positive

"-" means the relative change is negative

"*" means the relative change is less than 0.01% in magnitude

[2]**ROW will gain in welfare but lose in GDP**

[3]**The world will lose in welfare only when the mutual tariff rate is higher than 50%. When the mutual tariff is lower or equal to 50%, the world will witness a slight increase in welfare.**

elasticity value worsens the trade between China and the U.S., and diversify the trade to other economies. Dong and Whalley (2012)[11] presents the same finding for welfare and trade flows. Table 2.1 summarizes the findings of change in welfare or GDP under U.S.-China mutual trade war scenario by using CGE models. Papers are consistent to find that China and the world will be hurt by the trade spat despite some other countries gain slightly due to trade diversion.

Some other classes of the literature such as Qiao et al. (2019)[12] discussed implications of the trade war through literatures from political perspective, while Kim (2019)[20] offered a historical and social-economic analysis for trade war driving forces. Though those papers bring a much insight into the trade dispute, it's not the focus of this thesis that dealing with the international trade CGE modeling.

Chapter 3

Model

3.1 A Static CGE Prototype

The CGE prototype model (Hosoe, 2004)[16] encompasses all major parts of economic activity, and it focuses on the interrelationships of agents through the circular flow of income and international trade across countries. In this large-scale framework, producers produce by paying the wage to labor and the rental fee to capital. Households get wage and rental payments, then choose to consume and save, during which households and firms pay tax to the government who also consumes and saves. Unlike gravity analysis, which usually needs time series data, CGE models, to the contrary, may only require the data from a single period, often based on an input-output table.

This paper conducts an experiment within a standard CGE model by adding trade tariffs as an exogenous shock. The model is borrowed from Hosoe (2004)[16] and I modified utility and production functions, optimization expressions and parameters' calibration.

3.2 Components of A Standard CGE Model

3.2.1 Households

The aim of households is to maximize utility by choosing the optimal bundle of consumption goods X_l^h , subject to a budget constraint. The budget constraint description varies in papers such as fixed money supply, the sum of consumer's total expenditure from the previous period or the factor payments plus other sources of income. In this paper, it's regarded as the households' disposable income. The households' utility is expressed as a CES type of function to capture the assumption of constant elasticity of substitution between goods.

$$\max_{X_l^h} U_i(X_l^h) = [\sum_l \alpha_l^{\frac{1}{\sigma_i}} X_l^{h\frac{\sigma_i-1}{\sigma_i}}]^{\frac{\sigma_i}{\sigma_i-1}} \tag{3.1}$$

$$s.t. \quad \sum_s w_s FF_s - S^p - T^d = \sum_l P_l^d X_l^h$$

$i =$ country $\quad l =$ industry $\quad s =$ input facors

X_l^h : country i's household's consumption of good l $\quad P_l^d$: consumer price of good l $\quad T^d$: direct tax

S^p : household private saving $\quad w_s$: input factor s compensation $\quad FF_s$: input factors' endowments

α_l : share parameter of good l $\quad \sigma_i$: elasticity parameter of good l for each country i

3.2.2 Firms

The aim of firms is to maximize their profits subject to the constraint of the production technology whose property is captured by a CES technology function.

$$\max_{F_{sj}} \pi_j = p_j^s Y_j - \sum_s w_s F_{sj} \tag{3.2}$$

$$s.t. \quad Y_j = \phi_j (\sum_s \delta_{sj} (F_{sj})^{\rho_2})^{\frac{1}{\rho_2}}, \quad (\rho_2 = \frac{\sigma_j - 1}{\sigma_j})$$

l = industry on supply side s = input facors

π_j : the profit of firm j Y_j : value-added production of firm j

F_{sj} : input of factor s by the firm j p_j^s : supply price of product j

ϕ_j : production scale parameter δ_{sj} : factor s share parameter

σ_j : elasticity of technology subsitution

3.2.3 The Armington Aggregation

By the Armington assumption, exports and imports are not perfect substitutes, which means they are treated differently. One domestic supplied good in production can be substituted for more than one imported good. On the supply side, the composite of domestic supplied goods and imports are used to maximize the profit of composite good production.

$$\max_{M_l, D_l} \pi_l^q = p_l^q Q_l - p_l^m M_l - P_l^d D_l \tag{3.3}$$

$$s.t. \quad Q_l = \gamma_l (\alpha m_l (M_l)^{\rho_3} + \alpha d_l (D_l)^{\rho_3})^{\frac{1}{\rho_3}}, \quad (\rho_3 = \frac{\sigma_l - 1}{\sigma_l})$$

π_l^q : profit of composite good firm l p_l^q : price of composite good l Q_l : output of composite good l

D_l : input of domestic supplied good l M_l : input of imported supplied good l

$\alpha m_l, \alpha d_l$: share parameter of doemestic supplied and imported supplied good l

σ_l : elasticity parameter between domestic and imported supplied good γ_l : scale parameter

Firms produce domestically and export to foreign market to maximize the profit, taking the indirect tax into account.

$$\max_{E_l, D_l} \pi_l^z = p_l^e E_l + P_l^d D_l - \tau_l(1 + p_l^s) \cdot Z_l \tag{3.4}$$

$$s.t. \quad Z_l = \theta_l(\xi e_l(E_l)^{\rho_4} + \xi d_l(D_l)^{\rho_4})^{\frac{1}{\rho_4}}, \quad (\rho_3 = \frac{\sigma_l - 1}{\sigma_l})$$

π_l^z : the profit of gross output p_l^s : price of the supply side Z_l : total gross composite output

D_l : ouput to domesitc market good l E_l : output to foreign market good l

$\xi e_l, \xi d_l$: share parameter of doemestic output and to foreign market for good l

σ_l : elasticity parameter between dmestic and foreign market θ_l : scale parameter

3.2.4 Government

Government collects revenue through direct taxes on household incomes. The second source of the government revenue is indirect taxes on value-added production. Import tariffs collected from international trade is the third source of government income. Similar to households, government also consumes and saves up to its total income[1].

$$T_l = \tau_l \cdot p_l^s \cdot Z_l \tag{3.5}$$

[1]Equations (3.5) to (3.10) are directly borrowed from Nobuhiro (2004)[16] without modification.

$$T^d = \tau^d \cdot \sum_s w_s FF_s \tag{3.6}$$

$$T_l^m = \tau^{ml} \cdot p_l^m \cdot M_l \tag{3.7}$$

$$X_l^g = \frac{\mu_l}{p_l^q}(\sum_l T_l + \sum_l T_l^m + T^d - S^g) \tag{3.8}$$

T_l : indirect, value-added tax T^d : direct tax T_l^m : import tariff

τ_l : indirect tax rate τ^d : direct tax rate τ^{ml} : import tariff rate

X_l^g : government expenditure on good l μ_l : share of l good consumption by government

3.2.5 Saving

Public Saving

Government saves out of the income generated by indirect tax, direct tax and import tariff.

$$S^g = ss^g(\sum_l T_l + \sum_l T_l^m + T^d - S^g) \tag{3.9}$$

Private Saving

Households save out of their total income.

$$S^p = ss^p(\sum_s w_s FF_s) \tag{3.10}$$

ss^g : public saving rate ss^p : private saving rate,

3.2.6 Linkage

The linkage functions encompass each sector mentioned above.

Input-output Linkage (Intermediate input)

Firms not only use primary factors such as capital and labor for value-added production, they also use intermediate inputs to produce the final output. For example, the production of manufactured goods needs capital and labor, as well as other manufactured goods such as machinery and non-manufactured goods like electricity and transportation. The process of production cannot happen without manufactured and non-manufactured goods combining together. Also, because manufactured goods and non-manufactured goods are treated as tradable goods and non-tradable goods respectively mentioned by Li et al. (2018)[25], there is no substitutability between intermediate input factors, thus they are perfect complements. The input is therefore fixed by proportions. The fixed proportions are expressed as a, b respectively in a two-good scenario. $Q = min(\frac{X_1}{a}, \frac{X_2}{b}) = min(x_1, x_2) = x_j$, if $x_j \leq x_i$. Manufacturing goods and non-manufacturing goods are linked as perfect complements in intermediate input.

$$\max \pi_j = p_j^s Z_j - p_j^y Y_j - \sum_i p_i^q X_{ij} \tag{3.11}$$

$$s.t. \quad Z_j = min(\frac{X_{1j}}{a}, \frac{X_{2j}}{b}, \frac{Y_j}{c})$$

π_j : the profit of gross output of good j after value added and intermediate inputs

Y_j : value-added of good j Z_j : output of good j

X_{lj} : good l as intermediate inputs of producing good j

a, b, c : technologically determined constants

p_j^s, p_l^q, p_j^y : supply price, intermediate input price, value-added price

The nation's total investment is the sum of private saving, public saving and foreign saving. Foreign saving, as the net inflow of domestic currency from foreign markets, is also a country's trade surplus (deficit).

$$X_l^v = \frac{\lambda_l}{p_l^q} \cdot (S^p + S^g + \epsilon S^f) \tag{3.12}$$

S^p : Private saving S^g : government saving S^f : foreign saving

λ_l : share of invesmtnet on good l ϵ : exchange rate X_l^v : Total invesmtnet on good l

Countries' Linkage (International Trade)

The increase in tariff is assumed to be too small to influence the rest of the world price for any good if there are more than two countries involved in the simulation, which means that international trade won't change the prices from the country of origin. The linkage of international trade functions capture a country's value of exports, imports and trade surplus or deficit.

$$p_l^e = \epsilon p_l^{We} \tag{3.13}$$

$$p_l^m = \epsilon p_l^{Wm} \tag{3.14}$$

$$\sum_l p_l^{We} E_l + S^f = \sum_l p_l^{Wm} M_l \tag{3.15}$$

p_l^{We} : l good export price in foreign currecny p_l^e : l good export price in local currecny

p_l^{Wm} : l good import price in foreign currecny p_l^m : l good import price in local currecny

E_l : export of good l M_l : import of good l

Assumptions of the Standard CGE model

The prototype standard CGE model has some key assumptions:

1. Factors are fully mobile across industries. This assumption indicates that the same input factor always receives the same payment across industries l within a country. Thus w_s represents input factor payments rather than w_s^l. For more than one country, w_{si} is used. Markets push input factors to the industry that receives the highest compensation until the market converges to the equilibrium, under which circumstance, factors are not fully mobile. For example, whenever the wage in manufacturing sector is higher than the non-manufacturing sector, labor will shift to the manufacturing sector, increasing the labor supply and reducing the manufacturing job's wage, but decreasing the labor supply in non-manufacturing sector and increase non-manufacturing job's wage. The fully mobile input factors keep factor's payments the same across industries.

2. Fixed factor endowment. $\sum_l F_{sl} = FF_s$ This assumption says the total input of labor or capital won't change no matter how the economy changes. Due to the reason that it's a static model solving for a one-period change, the employment is assumed to be fixed in the short-run.

3. Input output equilibrium

$$Q_l = \gamma_l(\alpha m_l(M_l)^{\rho_3} + \alpha d_l(D_l)^{\rho_3})^{\frac{1}{\rho 3}} = X_l^h + X_l^g + X_l^v \tag{3.16}$$

This assumption indicates that to fit the general equilibrium, the supply of goods that are produced domestically or imported from other countries should equal the sum of each sector's consumption of that good, namely, household consumption, government consumption and private and public saving.

4. Balanced foreign saving.

The sum of every country's trade surplus equals to zero.

$$\sum_i S_i^f = 0 \tag{3.17}$$

5. Competitive market, zero profit assumption $\pi_j = 0$. All firms earn zero profit due to free entry. From equation (3.5),

$$\pi_j = p_j^s Z_j - p_j^y Y_j - \sum_i p_l^q X_{lj} = 0 \tag{3.18}$$

The structure of this CGE model used in my simulation is presented in Figure 3.1.

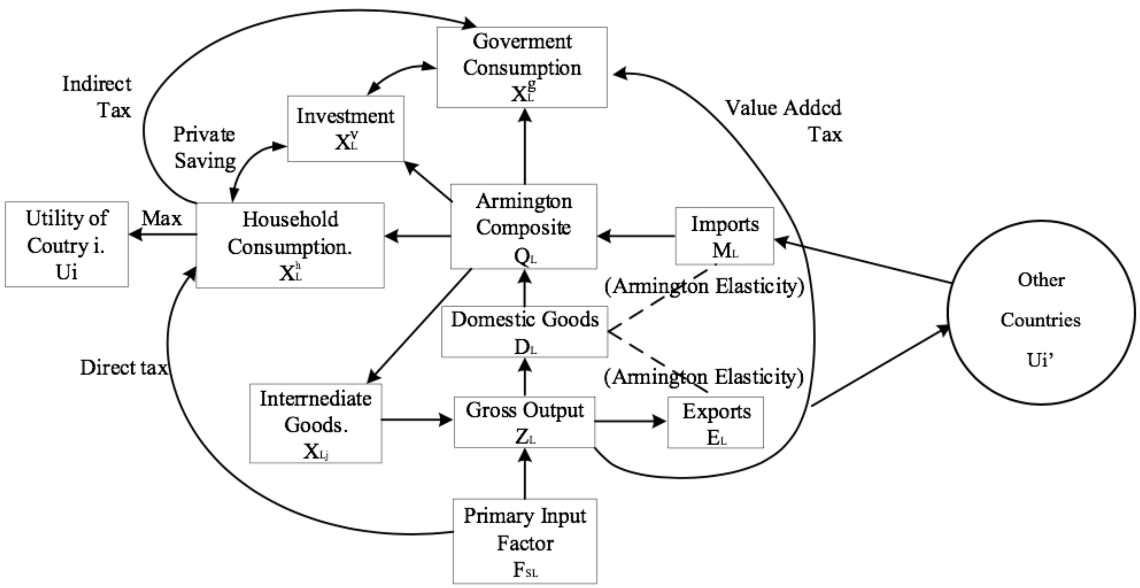

Figure 3.1: The Structure of a Standard CGE Model

3.3 Optimal Solutions of Model Equations

3.3.1 Households

The optimal households' consumption bundle is calculated by the following steps:

$$\max_{X_l^h} U_i(X_l^h) = [\sum_l \alpha_l^{\frac{1}{\sigma_i}} X_l^{h\frac{\sigma_i - 1}{\sigma_i}}]^{\frac{\sigma_i}{\sigma_i - 1}} \tag{3.19}$$

$$s.t. \quad \sum_s w_s FF_s - S^p - T^d = \sum_l P_l^d X_l^h$$

Lagrangian method is a commonly used approach to find an interior solution for a constrained optimization problem.

$$\mathcal{L}(X_l^h, \lambda) = U_i(X_l^h) + \lambda(\sum_{s=1}^{\infty} w_s FF_s - S^p - T^d - \sum_{l=1}^{\infty} P_l^d X_l^h) \tag{3.20}$$

$$\frac{\partial \mathcal{L}}{\partial X_l^h} = \frac{\partial U_i}{\partial X_l^h} - \lambda P_l^d = 0 \tag{3.21}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \sum_s w_s FF_s - S^p - T^d - \sum_l P_l^d X_l^h = 0 \tag{3.22}$$

from (3.21), the consumption of good l is expressed as,

$$\alpha_l^{\frac{1}{\sigma_i}} \frac{\sigma_i - 1}{\sigma_i} (X_l^h)^{\frac{-1}{\sigma_l}} = \lambda P_l^d \tag{3.23}$$

for another good l',

$$\alpha_{l'}^{\frac{1}{\sigma_i}} \frac{\sigma_i - 1}{\sigma_l} (X_{l'}^h)^{\frac{-1}{\sigma_i}} = \lambda P_{l'}^d \tag{3.24}$$

Take the ratio of (3.23) and (3.24)

$$\frac{P_l^d}{P_{l'}^d} = \frac{\alpha_l^{\frac{1}{\sigma_i}} (X_l^h)^{\frac{-1}{\sigma_i}}}{\alpha_{l'}^{\frac{1}{\sigma_i}} (X_{l'}^h)^{\frac{-1}{\sigma_i}}} \tag{3.25}$$

$$X_{l'}^h = \frac{\alpha_{l'}}{\alpha_l} \frac{P_l^d}{P_{l'}^d}^{\sigma_i} \cdot X_l^h \tag{3.26}$$

plug (3.26) into (3.22),

$$P_l^d X_l^h + P_{l'}^d \frac{\alpha_{l'}}{\alpha_l} \frac{P_l^d}{P_{l'}^d}^{\sigma_i} X_l^h + P_{l''}^d \frac{\alpha_{l''}}{\alpha_l} \frac{P_l^d}{P_{l''}^d}^{\sigma_i} X_l^h + ... = \sum_{s=1}^{\infty} w_s FF_s - S^p - T^d \tag{3.27}$$

The optimal consumption bundle of good l is derived as

$$X_l^h = \frac{\alpha_l (\sum_s w_s FF_s - S^p - T^d)}{(P_l^d)^{\sigma_i} [\sum_l \alpha_l (P_l^d)^{1-\sigma_i}]} \tag{3.28}$$

3.3.2 Firms

Firms' optimal input mix is calculated by the following steps:

$$\max_{F_{sj}} \pi_j = p_j^s Y_j - \sum_s w_s F_{sj} \tag{3.29}$$

$$s.t. \quad Y_j = \phi_j(\sum_s \delta_{sj}(F_{sj})^{\rho_2})^{\frac{1}{\rho_2}}, (\rho_2 = \frac{\sigma_j - 1}{\sigma_j})$$

Substitute the constraint into function (12),

$$\pi_j = p_j^s \phi_j(\sum_s \delta_{sj}(F_{sj})^{\rho_2})^{\frac{1}{\rho_2}} - \sum_s w_s F_{sj} \tag{3.30}$$

Take the first order condition

$$\frac{\partial \pi_j}{\partial F_{sj}} = p_j^s \phi_j(\sum_s \delta_{sj}(F_{sj})^{\rho_2})^{\frac{1-\rho_2}{\rho_2}} \delta_{sj}(F_{sj})^{\rho_2 - 1} - w_s = 0 \tag{3.31}$$

$$(F_{sj})^{1-\rho_2} = \frac{p_j^s (\phi_j)^{\rho_2} (Y_j)^{1-\rho_2}}{w_s} \tag{3.32}$$

The optimal demand of input factor s in firm j is

$$F_{sj} = (\frac{p_j^s \delta_{sj} (\phi_j)^{\rho_2} (Y_j)^{1-\rho_2}}{w_s})^{\frac{1}{1-\rho_2}} Y_j = (\frac{p_j^s \delta_{sj} (\phi_j)^{\frac{\sigma_j - 1}{\sigma_j}}}{w_s})^{\sigma_j} \cdot Y_j \tag{3.33}$$

3.3.3 The Armington Assumption

The optimal Armington composite factor is calculated by the following steps:

Input

The function below describes the profit maximization of production by using imported goods and domestic goods.

$$\max_{M_l, D_l} \pi_l^q = p_l^q Q_l - p_l^m M_l - P_l^d D_l \tag{3.34}$$

$$s.t. \quad Q_l = \gamma_l(\alpha m_l(M_l)^{\rho_3} + \alpha d_l(D_l)^{\rho_3})^{\frac{1}{\rho 3}}, (\rho_3 = \frac{\sigma_l - 1}{\sigma_l})$$

Plug the constraint into equation (3.34)

$$\pi_l^q = p_l^q \gamma_l(\alpha m_l(M_l)^{\rho_3} + \alpha d_l(D_l)^{\rho_3})^{\frac{1}{\rho 3}} - p_l^m M_l - P_l^d D_l \tag{3.35}$$

Take the first order condition

$$\frac{\partial \pi_l^q}{\partial M_l} = p_l^q \cdot \gamma_l \cdot \alpha m_l \cdot (M_l)^{\rho_3} + \alpha d_l \cdot (D_l)^{\rho_3})^{\frac{1}{\rho_3}} \cdot \alpha m_l \cdot (M_l)^{\rho 3 - 1} = p_l^m \tag{3.36}$$

$$(M_l)^{1-\rho_3} = \frac{\alpha m_l \cdot p_l^q \cdot (Q_l)^(1 - \rho_3) \cdot \gamma_l^{\rho_3}}{p_l^m} \tag{3.37}$$

Since M_l and D_l are symmetric, the optimal demand of composite good from imports and domestic supplied goods is

$$M_l = (\frac{\alpha m_l \cdot p_l^q \cdot \gamma_l^{\rho_3}}{p_l^m})^{\frac{1}{1-\rho_3}} \cdot Q_l \tag{3.38}$$

$$D_l = (\frac{\alpha d_l \cdot p_l^q \cdot \gamma_l^{\rho_3}}{P_l^d})^{\frac{1}{1-\rho_3}} \cdot Q_l \tag{3.39}$$

Output

The function below describes the profit maximization by selling in the domestic market or foreign markets.

$$\max_{D_l,E_l} \pi_l^z = p_l^e E_l + P_l^d D_l - \tau_l(1+p_l^s)\cdot Z_l \tag{3.40}$$

$$s.t. \quad Z_l = \theta_l(\xi e_l (E_l)^{\rho_4} + \xi d_l (D_l)^{\rho_4})^{\frac{1}{\rho_4}}$$

Similar to the above process, the optimal supply function of firm l to the domestic market and the foreign market respectively is derived as:

$$E_l = (\frac{\xi e_l \cdot p_l^s(1+\tau_l)\cdot \theta_l^{\rho_4}}{p_l^e})^{\frac{1}{1-\rho_4}} \cdot Z_l \tag{3.41}$$

$$D_l = (\frac{\xi d_l \cdot p_l^s(1+\tau_l)\cdot \theta_l^{\rho_4}}{P_l^d})^{\frac{1}{1-\rho_4}} \cdot Z_l \tag{3.42}$$

3.3.4 Intermediate Input

Firms use constant shares of intermediate input to maximize the profit of final outputs. The optimal demand of intermediate inputs is calculated below.

$$\max_{Z_j,Y_j,X_{lj}} \pi_j = p_j^s Z_j - p_j^y Y_j - \sum_i p_i^q X_{lj} \tag{3.43}$$

$$s.t. \quad Z_j = min(\frac{X_{1j}}{a}, \frac{X_{2j}}{b}, \frac{Y_j}{c})$$

$$X_{1j} = a\cdot Z_j \tag{3.44}$$

$$X_{2j} = b\cdot Z_j \tag{3.45}$$

$$Y_j = c \cdot Z_j \tag{3.46}$$

3.3.5 Competitive Market Zero Profit Condition

Plug equations (3.44) to (3.46) into equation (3.18) to get the price from the supply side.

$$\pi_j = p_j^s Z_j - p_j^y Y_j - \sum_l p_l^q X_{lj} = 0$$

$$p_j^s Z_j = p_j^y \cdot c \cdot Z_j + p_1^q \cdot a \cdot Z_j + p_2^q \cdot b \cdot Z_j \tag{3.47}$$

$$p_j^s = p_1^q \cdot a + p_2^q \cdot b + p_j^y \cdot c \tag{3.48}$$

3.4 Model Solving Procedure

3.4.1 Sets, Variables and Parameters

Sets

Sets define the domain of the model, defining where each component of the equations (3.39) to (3.76) belongs. In the set, there are $U.S., China...ROW$, i countries in the model, and manufactured and non-manufactured industries l are used to represent tradable goods and non-tradable goods for aggregation; labor and capital s are the only primary factors used for input. Thus the domain in this model is written as s, l, i in the $s \times l \times i$ database. Manufacturing and non-manufacturing goods as production and consumption, capital and labor as primary input factors, the U.S., China and ROW as countries, the database domain is written in $2 \times 2 \times 3$. To express the interaction within sectors, different notations are used for the same sector but in different types. l, j express the interaction between industries, and the trade between a pair of countries is expressed as i, r.

Country: i, r Input factor: s Industry: l, j

Parameters

When solving the equation system, parameters are defined as exogenous variables, scale parameters, share parameters and elasticity parameters.

Exogenous Variables: $a, b, c, \tau_l, \tau^d, \tau^{ml}, \mu_l, \lambda_l, T_l, T_d, T_l^m, ss^g, ss^p, p_l^{We}, p_l^{Wm}$

Scale Parameters: $\phi_j, \gamma_l, \theta_l$

Share Parameters: $\alpha_l, \delta_{sj}, \alpha m_l, \alpha d_l, \xi e_l, \xi d_l, \mu_l, \lambda_l$

Elasticity Parameters: $\sigma_l, \sigma_j, \rho, \rho_3, \rho_4$

An exogenous shock is an unexpected or unpredictable event that affects an economy. τ^{ml}, the import tariff rate, is modified to create exogenous shock.

Variables

Endogenous variables are solved within the equation system through utility optimization, and the simulation result is based on the percentage change of these variables.

Endogenous Variables: $X_{lj}, F_{sl}, X_l^h, X_l^g, X_l^v, Y_l, Z_l, Q_l, E_l, M_l, D_l, w_s, P_l^d, p_l^q,$
$p_l^s, p_l^y, p_l^e, p_l^m, T_l, T_l^m, T^d, S^p, S^g, \epsilon$

Numeraire Variable: w_{labor}

The impact of this CGE model is described in the form of relative change which removes nominal concerns. Thus, one price variable, w_{labor} (labor input compensation) is fixed at the bench mark value "1", and the change of other price variables are measured against this numeraire value.

3.4.2 Variables and Equations Square

In this non-linear system, the goal is to maximize the objective functions (3.49) and (3.50) with the same number of equations and endogenous variables whose are solved after the optimization process.

Objective Function Utility Maximization

$$\mathbf{max}\ U_i(X_l^h) = [\sum_{l=1}^{\infty} \alpha_l^{\frac{1}{\sigma_l}} X_l^{h^{\frac{\sigma_l - 1}{\sigma_l}}}]^{\frac{\sigma_l}{\sigma_l - 1}} \tag{3.49}$$

$$\mathbf{max}\ \sum_i U_i \tag{3.50}$$

For Each Country i

Equations (3.51) to (3.76) are equations used to solve for the same number of endogenous variables that are extracted from (3.1) to (3.48):

Households

$$X_l^h = \frac{\alpha_l(\sum_s w_s FF_s - S^p - T^d)}{(P_l^d)^{\sigma_i}[\sum_l \alpha_l (P_l^d)^{1-\sigma_i}]} \tag{3.51}$$

Production

Value-added production technology function

$$Y_j = \phi_j(\sum_s \delta_{sj}(F_{sj})^{\rho_2})^{\frac{1}{\rho_2}} \tag{3.52}$$

Optimal Demand of primary factor through optimization

$$F_{sj} = (\frac{p_j^s \delta_{sj} (\phi_j)^{\frac{\sigma_j - 1}{\sigma_j}}}{w_s})^{\sigma_j} \cdot Y_j = (\frac{p_j^s \delta_{sj}}{w_s})^{\sigma_j} \cdot \phi_j^{\sigma_j - 1} \cdot Y_j \tag{3.53}$$

Leontif intermediate input functions

$$X_{1j} = a \cdot Z_j \tag{3.54}$$

$$X_{2j} = b \cdot Z_j \tag{3.55}$$

$$Y_j = c \cdot Z_j \tag{3.56}$$

Zero profit Condition

$$p_j^s = p_1^q \cdot a + p_2^q \cdot b + p_j^y \cdot c \tag{3.57}$$

Armington Aggregation

Input aggregation of imports and domestic goods

$$Q_l = \gamma_l (\alpha m_i (M_l)^{\rho_3} + \alpha d_l (D_l)^{\rho_3})^{\frac{1}{\rho 3}} \tag{3.58}$$

$$M_l = (\frac{\alpha m_l \cdot p_l^q \cdot \gamma_l^{\rho_3}}{(1 + \tau^{ml}) p_l^m})^{\frac{1}{1-\rho_3}} \cdot Q_l \tag{3.59}$$

$$D_l = (\frac{\alpha d_l \cdot p_l^q \cdot \gamma_l^{\rho_3}}{P_l^d})^{\frac{1}{1-\rho_3}} \cdot Q_l \tag{3.60}$$

Output disaggregation to domestic and export goods

$$Z_l = \theta_l (\xi e_l (E_l)^{\rho_4} + \xi d_l (D_l)^{\rho_4})^{\frac{1}{\rho_4}}, (\rho_3 = \frac{\sigma_l - 1}{\sigma_l}) \tag{3.61}$$

$$E_l = (\frac{\xi e_l \cdot p_l^s(1+\tau_l) \cdot \theta_l^{\rho_4}}{p_l^e})^{\frac{1}{1-\rho_4}} \cdot Z_l \tag{3.62}$$

$$D_l = (\frac{\xi d_l \cdot p_l^s(1+\tau_l) \cdot \theta_l^{\rho_4}}{P_l^d})^{\frac{1}{1-\rho_4}} \cdot Z_l \tag{3.63}$$

Government

$$T_l = \tau_l \cdot p_l^s \cdot Z_l \tag{3.64}$$

$$T^d = \tau^d \cdot \sum_s w_s FF_s \tag{3.65}$$

$$T_l^m = \tau^{ml} \cdot p_l^m \cdot M_l \tag{3.66}$$

$$X_l^g = \frac{\mu_l}{p_l^q}(\sum_l T_l + \sum_l T_l^m + T^d - S^g) \tag{3.67}$$

Saving

$$S^g = ss^g(\sum_l T_l + \sum_l T_l^m + T^d - S^g) \tag{3.68}$$

$$S^p = ss^p(\sum_s w_s FF_s) \tag{3.69}$$

Linkage

$$X_l^v = \frac{\lambda_l}{p_l^q} \cdot (S^p + S^g + \epsilon S^f) \tag{3.70}$$

$$p_l^e = \epsilon p_l^{We} \tag{3.71}$$

$$p_l^m = \epsilon p_l^{Wm} \tag{3.72}$$

$$\sum_l p_l^{We} E_l + S^f = \sum_l p_l^{Wm} M_l \tag{3.73}$$

$$Q_l = X_l^h + X_l^g + X_l^v \tag{3.74}$$

$$\sum_l F_{sl} = FF_s \tag{3.75}$$

Zero-sum Foreign Saving

$$\sum_i S_i^f = 0 \tag{3.76}$$

To solve the non-linear constrained optimization system, exactly the same number of equations and endogenous variables are needed in the system. For each country, there are $(l^2+l\times h+18\times l+4)\times i+1$ variables and $(l^2+l\times h+18\times l+4)\times i$ equations from (3.51) to (3.76), so that there's one redundant equation. However, Walras's Law states that if the economy is in general equilibrium, the excess supply will match the excess demand. If there are n markets in the economy and the 1^{st} to the $(n-1)^{th}$ markets are all in equilibrium, the n^{th} market must also be in equilibrium condition to fit general equilibrium. This application is used in equation (3.76), the zero-sum foreign saving. In other words, under global general equilibrium, if every country fits the general equilibrium condition, the sum of each country's foreign saving or trade surplus (deficit) should always be zero. Thus in our model, under the domain of $2\times 2\times i$, each country has $4+4+36+4=48$ equations and variables. And the Walras's Law foreign saving equation encompasses the whole economy which is automatically satisfied under general equilibrium.

Chapter 4

Data

A CGE model describes the whole economy and every sector's interaction, and the data that modelers collect is the outcome of the general equilibrium from the last period. By the Armington assumption, goods are classified as tradable and non-tradable goods. The data set should include production, consumption, household, government, investment and trade, in particular:

1. The quantity of each type of production, the input value and the household compensation

2. The quantity of goods each sector consumes or being used as intermediate inputs.

3. The saving of each sector

4. The interaction between countries.

A database is needed to include all the above information before running the simulation.

4.1 Social Accounting Matrix Database

The method of collecting data for a CGE model is to construct a Social Accounting Matrix (SAM) based on the input-output files (I/O). A Social Accounting Matrix also includes the same components: production activities, input factors and institutions. In this matrix, the payment that each sector receives should exactly match the amount that it transfers out. To describe international trade, the total sum of the export of any commodity from all countries should equal to the total sum of its import from all countries to fit the general equilibrium. Otherwise, the redundant output makes the input won't be fully consumed, and results in unbalanced CGE database which will induce an error in the computation process.

When constructing the SAM table, I choose 2014 as the base year and treat the data as the outcome of general equilibrium. The file that this paper uses is the 2014 database which is the most up-to-date World Input-Output Database published (WIOD) , and its Socio Economic Accounts Release 2016[35], published in February 2018 for value-added input. The data on intermediate input activities was aggregated from 56 sectors are classified according to the International Standard Industrial Classification revision 4. The trade and taxation related data is collected from the World Bank Open Data, and the base year is 2014. Since I can't find the share of government's manufactured and non-manufactured consumption, I assume it's the same distribution as household's consumption distribution. Three of the entries "Government-Household", "Investment-Household" and "Investment-Government" are adjusted according to other data in the table. For the reason that all other columns and rows add up to the same number, if two of the above three entries are defined, the third entry will come out spontaneously from the table. This is another applications of Walras' law that in the general equilibrium, if there are n markets and from the first to the $n-1$th market is all in equilibrium , the nth market is in its equilibra per se.

In addition, if only two countries are considered, U.S. and China, they must represent the world. This means that each country must import the same amount of the good that the other country exports. In order to follow this constraint, I modified one of the least influential pairs of entries, the Chinese exports and the U.S. imports. Manufactured exports take up 94% of the total exports in 2014, and I adjusted this number to 94.2% to balance the trade. However, the modification of U.S. manufactured imports has a larger percentage change than that of China.

The unit of entries is adjusted to the U.S. dollars by the 2014 exchange rate. For the ease of entries, I scale the data to 10^5 based on the World Input-Output File, which is measured in millions of dollars. Thus, the unit in all of entries is $10^5 \cdot 10^6 = 10^{11}$ USD. The first step is to check if the sum of all entries in the column equals to the sum of corresponding row entries. Secondly, the table's reliability can be checked by calculating the value-added to economic indicators. For example, the United States GDP in 2014 is 17.5 trillion dollars which is very close to the SAM matrix entries, the sum of value-added output, $20.8 + 153 = 173.8$ times 10^{11} USD. The same method can be applied to other entries for all countries.

The Social Accounting Matrix database for simulation is presented in the Table

4.1 and 4.2.

M = manufactured good, NM = non-manufactured good, K = Capital, L = Labor

Table 4.1: United States in 2 Country Simulation

U.S.	10^{11} USD	Activities		Factors		Institutions					Foreign	Total
		M	NM	K	L	Household	Government	Investment	Indirect Tax	Import Tariff	Export	
Activities	M	16	15			18	4	12.73			15	80.73
	NM	18	73			104	22	16.52			9	242.52
Factors	K	11	65									76
	L	9.8	88									97.8
Value Added		20.8	153									
Institutions	Household			76	97.8							173.8
	Government					41.77			0	0.5		42.27
	Investment					10.03	16.27				2.95	29.25
	IDT	0	0									0
	TRF	0.38	0.12									0.5
Domestic Input		54.8	241									
Foreign	Import	25.55	1.4									26.95
Total		80.73	242.52	76	97.8	173.8	42.27	29.25	0	0.5	26.95	

Table 4.2: China in 2 Country Simulation

China	10^{11} USD	Activities		Factors		Institutions					Foreign	Total
		M	NM	K	L	Household	Government	Investment	Indirect Tax	Import Tariff	Export	
Activities	M	78	32			5.6	2	24.22			25.55	167.37
	NM	39	44			32	11	34.8			1.4	162.2
Factors	K	17	29									46
	L	13	43									56
Value Added		30	72									
Institutions	Household			46	56							102
	Government					28			9.6	0.97		38.57
	Investment					36.4	25.57				-2.95	59.02
	IDT	4.55	5.05									9.6
	TRF	0.82	0.15									0.97
Domestic Input		151.55	153.05									
Foreign	Import	15	9									24
Total		167.37	162.2	46	56	102	38.57	59.02	9.6	0.97	24	

4.2 Calibration

As one of the most important features of CES utility function, elasticity parameters are usually plugged in exogenously from econometric analysis or from previous literature to take up the redundant degree of freedom. They determine the responsiveness of consumers or producers due to the change in price or income. Also, it tells how hard it is to compensate one unit loss of one good keeping the same utility. Elasticity parameters are then used to calibrate the share parameters and the scaling parameters in the model. This process is called calibration where modelers solve the optimization with numerically defined functions, then should the endogenous result replicate the plugged-in data.

4.2.1 Method

All parameters that needs to be calibrated are listed on Table 4.3.

Elasticity Parameters

Table 4.3: Parameter Table

	Scale	Share	Elasticity
Utility Function		α_l	σ_i, ρ_1
Production Function	ϕ_j	δ_{sj}	σ_j, ρ_2
Armington Input	γ_l	$\alpha m_l, \alpha d_l$	σ_l^q, ρ_3
Armington Output	θ_l	$\xi e_l, \xi d_l$	σ_l^z, ρ_4

Elasticity parameters are calibrated through the rearrangement of optimized expressions.

σ_i : elasticity of subsitution in country i's utility function

$$\sigma_i = \frac{d(X_l^h/X_{l'}^h)}{X_l^h/X_{l'}^h} \bigg/ \frac{d(P_l^d/P_{l'}^d)}{P_l^d/P_{l'}^d}$$

$$\rho_1 = \frac{\sigma_i - 1}{\sigma_i}$$

σ_j : elasticity of subsitution between input factor s in industry j in production function

$$\sigma_j = \frac{d(F_{sj}/F_{s'j})}{F_{sj}/F_{s'j}} \Big/ \frac{d(w_s/w_{s'})}{w_s/w_{s'}} = \frac{d(K_j/L_j)}{K_j/L_j} \Big/ \frac{d(R/W)}{R/W}$$

$$\rho_2 = \frac{\sigma_j - 1}{\sigma_j}$$

σ_l^q : elasticity of subsitution between imported and domestic supply in producing l

$$\sigma_l^q = \frac{d(M_l/D_l)}{M_l/D_l} \Big/ \frac{d(p_l^m/p_l^d)}{p_l^m/p_l^d}$$

$$\rho_3 = \frac{\sigma_l^q - 1}{\sigma_l^q}$$

σ_l^z : **elasticity of subsitution between exports and domestic ouput of l**

$$\sigma_l^z = \frac{d(E_l/D_l)}{E_l/D_l} \Big/ \frac{d(p_l^e/p_l^d)}{p_l^e/p_l^d}$$

$$\rho_4 = \frac{\sigma_l^z + 1}{\sigma_l^z}$$

When σ Approaches 1

It's worth noting that when the elasticity parameter equals to one, the CES function converges to Cobb-Douglas function $U = \prod (X_l^h)^{\alpha_l}$ that often appears in the text books and the original version of the standard static CGE model in (Hosoe, 2004)[16].
[1]

[1]See Appendix C for the GAMS code

Share Parameters

Share parameters express the preference over goods: the percentage of distribution to optimize under constraint.

Household Utility Function Consumption Share α_l

$$\sum_l \alpha_l = 1$$

$$\frac{\alpha_l}{\alpha_{l'}} = \frac{X_l^h \cdot (P_l^d)^{\sigma_i}}{X_{l'}^h \cdot (P_{l'}^d)^{\sigma_i}}$$

$$\alpha_l = \frac{X_l^h \cdot (P_l^d)^{\sigma_i}}{\sum_l X_l^h \cdot (P_l^d)^{\sigma_i}} \tag{4.1}$$

Through the similar process, the input factor share, the shares of foreign goods, domestic goods and foreign goods in the Armington composite functions can be solved.

$$(\delta_{sj})^{\frac{1}{\sigma_l}} = \frac{FF_{sj} \cdot (w_s)^{\sigma_l}}{\sum_s FF_{sj} \cdot (w_s)^{\sigma_l}}$$

$$\delta_{sj} = \frac{FF_{sj}^{\frac{1}{\sigma_l}} \cdot w_s}{\sum_s FF_{sj}^{\frac{1}{\sigma_l}} \cdot w_s} = \frac{FF_{sj}^{(1-\rho_2)}) \cdot w_s}{\sum_s FF_{sj}^{(1-\rho_2)} \cdot w_s} \tag{4.2}$$

Armington Composite Share in function 3.58 and 3.6

$$\alpha m_l = \frac{M_l^{\frac{1}{\sigma_l^q}} \cdot p_l^m}{M_l^{\frac{1}{\sigma_l^q}} \cdot p_l^m + D_l^{\frac{1}{\sigma_l^q}} \cdot p_l^m} = \frac{M_l^{(1-\rho_3)} \cdot p_l^m}{M_l^{(1-\rho_3)} \cdot p_l^m + D_l^{(1-\rho_3)} \cdot P_l^d} \tag{4.3}$$

$$\alpha d_l = \frac{D_l^{\frac{1}{\sigma_l^q}} \cdot P_l^d}{M_l^{\frac{1}{\sigma_l^q}} \cdot p_l^m + D_l^{\frac{1}{\sigma_l^q}} \cdot P_l^d} = \frac{D_l^{(1-\rho_3)} \cdot p_l^m}{M_l^{(1-\rho_3)} \cdot p_l^m + D_l^{(1-\rho_3)} \cdot P_l^d} \tag{4.4}$$

$$\xi e_l = \frac{E_l^{\frac{1}{\sigma_l^z}} \cdot p_l^e}{E_l^{\frac{1}{\sigma_l^z}} \cdot p_l^e + D_l^{\frac{1}{\sigma_l^z}} \cdot P_l^d} = \frac{E_l^{(1-\rho_4)} \cdot p_l^e}{E_l^{(1-\rho_4)} \cdot p_l^e + D_l^{(1-\rho_4)} \cdot P_l^d} \tag{4.5}$$

$$\xi d_l = \frac{D_l^{\frac{1}{\sigma_l^z}} \cdot p_l^e}{E_l^{\frac{1}{\sigma_l^z}} \cdot p_l^e + D_l^{\frac{1}{\sigma_l^z}} \cdot P_l^d} = \frac{D_l^{(1-\rho_4)} \cdot p_l^e}{E_l^{(1-\rho_4)} \cdot p_l^e + D_l^{(1-\rho_4)} \cdot P_l^d} \tag{4.6}$$

Scale Parameters

Scale parameter of the value-added production, rearranged from equation (3.52)

$$\phi_j = Y_j / (\sum_s \delta_{sj} (F_{sj})^{\rho_2})^{\frac{1}{\rho_2}}$$

Scale parameter of Armington input composite function, rearranged from equation (3.58)

$$\gamma_l = Q_l / (\alpha m_l (M_l)^{\rho_3} + \alpha d_l (D_l)^{\rho_3})^{\frac{1}{\rho_3}}$$

Scale parameter of Armington output composite function, rearranged from equation (3.61)

$$\theta_l = Z_l / (\xi e_l (E_l)^{\rho_4} + \xi d_l (D_l)^{\rho_4})^{\frac{1}{\rho_4}}$$

4.2.2 Calibration Table

The calibration result of parameters based on 2 country's Social Accounting Matrix is given in the Table 4.4, 4.5 and 4.6. The value of consumption and input technology transformation elasticities are set as 0.5 and 1.25 respectively.

M = manufactured good   NM = non-manufactured good

L = labor   K = capital

D = domestic good   IM = import good   E = export good

Table 4.4: Utility and Production Share Parameters α_l,δ_{sj}

	Utility Share		Production Share			
Country	M	NM	K-M	L-M	K-NM	L-NM
U.S.	0.148	0.852	0.514	0.486	0.462	0.538
China	0.149	0.851	0.533	0.467	0.451	0.549

Table 4.5: Armington Share Parameters α_{ml}, α_{dl}, ξ_{el}, ξ_{dl}

	Armington Input Share				Armington Ouput Share			
Country	D-M	IM-M	D-NM	IM-NM	M-D	M-E	NM-D	NM-E
U.S.	0.552	0.448	0.922	0.078	0.380	0.620	0.165	0.835
China	0.733	0.267	0.801	0.199	0.310	0.690	0.088	0.912

Table 4.6: Scale Parameters ϕ_j, γ_l, θ_l

	Production		Armington Input		Armington Output	
Country	M	NM	M	NM	M	NM
U.S.	1.997	1.982	1.991	1.168	2.119	3.373
China	1.986	1.970	1.653	1.468	2.250	4.915

Chapter 5

Results

5.1 Computation Procedure

Under the standard general equilibrium, six different tariff rate scenarios are simulated. The first scenario is the U.S. imposes a unilateral 25% tariff on Chinese imports, and China doesn't retaliate. Then the second scenario simulates China retaliating by a 25% on the U.S. imports, signals the start of the bilateral trade war. U.S. imposes 50% tariff on Chinese imports and China keeps the 25% tariff in the third scenario. China retaliated with a 50% tariff on the U.S. imports on the fourth scenario. The U.S. adds a 100% tariff, then China retaliates by a 100% tariff are the fifth and the sixth scenario.

The computation of the U.S.-China trade war simulation is run on General Algebraic Modeling System (GAMS)[1]. GAMS is a modeling system for mathematical programming and optimization, and it's specifically designed for complex, large scale

[1] See Appendix B for the complete GAMS code for simulation

non-linear optimization problem. The solving command, **NLP**, is the short form of non-linear programming, designed for solving smooth functions and non-discrete variables. In this large scale general equilibrium model, the aim is to maximize the sum of countries' utility, and the code goes through the process of:

1. Define sets
2. Construct Social Accounting Matrix database.
3. Plug in value for parameters and exogenous value
3. List parameters and variables
4. Calibrate the elasticity, share and scale parameters
5. Construct the system of variables and equations
6. Solve the system to replicate the base year data
7. Add exogenous shock and solve the model one more time
8. Conduct the percentage change of endogenous variables.

The Table 5.1 shows the percentage change of welfare, manufacturing and non-manufacturing production, labor employment and trade. As the same as Chunding Li et al. (2018)[25], the relative change of labor employment is represented by the change in manufacturing sector. It's assumed that the total employment is fixed, so that the sign of change in employment of manufacturing sector is always opposite to that of the non-manufacturing sector. Also, there's no entry of world total production because the gain/loss of labor in the manufacturing sector will be compensated by the non-manufacturing sector, sum of the total production won't change by adding exogenous tariff shock. For the relative change in trade, only"import" is included rather than both "import" and "export" due to the reason that in 2 country scenario, each country's export should match up the other's county's import plus foreign saving, and in a static model, saving is plugged in as a parameter (exogenous variable), thus the change of export is the same as the change of the other country's import.

Since the elasticity parameters influence the utility, production and Armington composite functions' magnitude, to run the simulation, they were set with the suggested values: substitution elasticity between manufactures and non-manufactures goods as 0.5 (Dong and Whalley, 2012)[11], substitution elasticity between capital and labor as 1.25 (karabarbounis et al., 2013)[19]. The value of 0.7 suggested by Muck et al.(2017)[31] will also be tested. Lastly, the Armington elasticities are set as 2 (Hertel et al., 2014)[13]. Since most papers conduct Armington elasticity test rather than the elasticities of utility and production functions, thus values are chosen for the top consumption substitution and input factor substitution to conduct sensitivity check in this paper[2].

5.2 Simulation Results

[2]Sensitivity check is provided in Results Chapter and Appendix C

Table 5.1: 2 Country Simulation Result, $\sigma_i = 0.5 \quad \sigma_j = 1.25 \quad \sigma_l^q = \sigma_l^z = 2$

	U.S. Unilateral Tariff War (25%)					U.S. China Mutual Tariff War (25%)				
Country	**Welfare**	**M**	**NM**	**Employment-M**	**Import**	**Welfare**	**M**	**NM**	**Employment-M**	**Import**
U.S.	-1.42%	-3.19%	0.43%	-3.27%	-3.29%	-3.17%	1.06%	-0.14%	1.09%	-17.09%
China	-2.11%	0.42%	-0.17%	0.47%	-14.91%	-3.58%	-2.43%	1.01%	-2.70%	-16.47%
World	-1.58%	-1.06%	0.24%	-1.14%	-8.76%	-3.26%	-1.00%	0.23%	-1.07%	-16.80%

	U.S. imposes 50% tariff, China imposes 25% tariff					U.S. imposes 50% tariff, China imposes 50% tariff				
Country	**Welfare**	**M**	**NM**	**Employment-M**	**Import**	**Welfare**	**M**	**NM**	**Employment-M**	**Import**
U.S.	-4.12%	-1.05%	0.14%	-1.07%	-19.13%	-5.46%	2.45%	-0.33%	2.51%	-28.69%
China	-5.19%	-1.94%	0.81%	-2.15%	-27.27%	-6.23%	-3.78%	1.58%	-4.19%	-28.55%
World	-4.37%	-1.57%	0.35%	-1.69%	-22.96%	-5.64%	-1.23%	0.28%	-1.31%	-28.62%

	U.S. imposes 100% tariff, China imposes 50% tariff					U.S. imposes 100% tariff, China imposes 100% tariff				
Country	**Welfare**	**M**	**NM**	**Employment-M**	**Import**	**Welfare**	**M**	**NM**	**Employment-M**	**Import**
U.S.	-6.64%	-0.19%	0.03%	-0.20%	-30.95%	-8.58%	5.33%	-0.73%	5.48%	-43.33%
China	-8.59%	-2.81%	1.17%	-3.12%	-43.30%	-9.95%	-4.98%	2.07%	-5.51%	-45.18%
World	-7.10%	-1.74%	0.39%	-1.86%	-36.77%	-8.91%	-0.76%	0.17%	-0.79%	-44.20%

Table 5.2: 2 Country Simulation Result, $\sigma_i = 0.5 \quad \sigma_j = 0.7 \quad \sigma_l^q = \sigma_l^z = 2$

	U.S. Unilateral Tariff War (25%)					U.S. China Mutual Tariff War (25%)				
Country	**Welfare**	**M**	**NM**	**Employment-M**	**Import**	**Welfare**	**M**	**NM**	**Employment-M**	**Import**
U.S.	-1.42%	-3.19%	0.43%	-3.27%	-3.29%	-3.16%	1.06%	-0.14%	1.08%	-17.09%
China	-2.11%	0.42%	-0.17%	0.46%	-14.92%	-3.59%	-2.43%	1.01%	-2.69%	-16.47%
World	-1.58%	-1.06%	0.24%	-1.14%	-8.76%	-3.26%	-1.00%	0.23%	-1.07%	-16.80%

	U.S. imposes 50% tariff, China imposes 25% tariff					U.S. imposes 50% tariff, China imposes 50% tariff				
Country	**Welfare**	**M**	**NM**	**Employment-M**	**Import**	**Welfare**	**M**	**NM**	**Employment-M**	**Import**
U.S.	-4.20%	-1.05%	0.14%	-1.07%	-19.12%	-5.46%	2.44%	-0.33%	2.51%	-28.69%
China	-5.20%	-1.93%	0.81%	-2.15%	-27.27%	-6.23%	-3.78%	1.57%	-4.18%	-28.54%
World	-4.44%	-1.57%	0.35%	-1.69%	-22.96%	-5.64%	-1.23%	0.28%	-1.31%	-28.62%

	U.S. imposes 100% tariff, China imposes 50% tariff					U.S. imposes 100% tariff, China imposes 100% tariff				
Country	**Welfare**	**M**	**NM**	**Employment-M**	**Import**	**Welfare**	**M**	**NM**	**Employment-M**	**Import**
U.S.	-6.64%	-0.20%	0.03%	-0.20%	-30.94%	-8.58%	5.32%	-0.72%	5.48%	-43.33%
China	-8.60%	-2.80%	1.17%	-3.11%	-43.30%	-9.95%	-4.97%	2.07%	-5.50%	-45.18%
World	-7.10%	-1.74%	0.39%	-1.86%	-36.76%	-8.91%	-0.75%	0.17%	-0.78%	-44.20%

When the elasticity parameters are set as $\sigma_i = 0.5, \quad \sigma_j = 1.25, \quad \sigma_l^q = \sigma_l^z = 2$. Firstly, the simulation results show that U.S. will be hurt by a loss in welfare, and the trade volume, and the loss enlarges if the U.S. impose higher tariff no matter what trade strategy China employ. When the rise of import tariff is unilateral or comparatively higher in the U.S., the employment in manufacturing sector will decrease, whose loss is less significant when tariff escalates. On the other hand, when China and the U.S. raise the import tariff by same value, the U.S. will gain in the manufacturing production and employment, in addition, both indicators will witness a larger gain if the tariff rate surges. The manufacturing production will increase by 1.06%, 2.45% and 5.33% under a mutual tariff of 25%, 50% and 100% respectively.

Secondly, China is expected to be hurt in all indicators except non-manufacturing production when the U.S. import tariff is equal to or more than 50%, and the loss of China is expected to be more severe than the loss of the U.S. in all indicators. Moreover, if China chooses to retaliate engaging in a mutual tariff war, The loss will be more significant. However, if China chooses status-quo and no retaliation, though the China's welfare will decrease by -2.11%, 0.69% more than that of the U.S., it's interesting to see that China may experience a gain in the manufacturing sector. From the simulation result, under the no-retaliation situation, China's capital price is expected to be 0.067% higher than the labor price. Lower labour price may induce a higher employment rate and the increase in manufacturing production slightly by 0.42%. Adversely, the volume of the U.S.'s exports decrease more than the Chinese exports. This phenomenon might be due to the reason that higher tariff rates push up the price in the U.S. market, and in the long run, consumers in China prefers domestic products because of their lower price.

From the world's perspective, all indicators present a negative change, and the global trade is expected to be hurt the most. Under 100% mutual tariff war, the volume of trade decreases by 44.20%. Despite the U.S. gains in the manufacturing sector under mutual tariff war and higher tariff rates, the loss of China outweighs the gain of the U.S.. The world manufacturing sector total production is expected to reduce in all scenarios. Thus, in the world of two countries, the U.S. and China trade spat hurts the global economy.

The table 5.2 shows the sensitivity of elasticity of technology transformation. When the value is set as 0.7, the lower bound value that I found in the recent literature, the relative change is subtle compared to the 1.25 elasticity, and most of the change is within the range of 0.01%. This result might be caused by the calibration of share parameters δ_{sj} that the capital input and the labor input share in manufacturing and non-manufacturing sectors are close to 50%, thus, both input factors are given similar preference in both sectors.

Table 5.3 and 5.4 shows the elasticity of substitution between manufactured goods and non-manufactured goods is set as 1 and 2 respectively. When the elasticity parameter equals to one, the function CES utility function converts to a Cobb-Douglas utility function [3]. The experiment of elasticity of 2 is based on the Arminton assumption, treating manufactured goods as tradable goods, non-manufactured goods as non-tradable goods, then applying "the rule of 2" (Dong ans Whalley, 2012) [11].

[3] See Appendix D for detailed explanation

When the elasticity of substitution value is large, the welfare loss due to tariff war is relatively small, which result might be caused by the better substitutability between different goods that compensate the original loss. However, higher σ_i hurts the U.S. manufacturing sector where gains when $\sigma_i = 0.5$ and same tariff rates are imposed from both sides. But if $\sigma_i = 1$, only 100% tariff strategy can help the U.S. manufacturing, and the manufacturing gets continually worse off under higher tariff if $\sigma_i = 2$.

Table 5.3: 2 Country Simulation Results, $\sigma_i = 1 \quad \sigma_j = 0.7 \quad \sigma_l^q = \sigma_l^z = 2$

	U.S. Unilateral Tariff War (25%)					**U.S. China Mutual Tariff War (25%)**				
Country	**Welfare**	**M**	**NM**	**Employment-M**	**Import**	**Welfare**	**M**	**NM**	**Employment-M**	**Import**
U.S.	-1.38%	-3.85%	0.52%	-3.94%	-3.41%	-3.04%	-0.71%	0.10%	-0.73%	-17.33%
China	-2.16%	0.38%	-0.16%	0.42%	-15.31%	-3.72%	-2.48%	1.03%	-2.75%	-17.46%
World	-1.56%	-1.35%	0.31%	-1.46%	-9.01%	-3.20%	-1.75%	0.40%	-1.88%	-17.39%

	U.S. imposes 50% tariff, China imposes 25% tariff					**U.S. imposes 50% tariff, China imposes 50% tariff**				
Country	**Welfare**	**M**	**NM**	**Employment-M**	**Import**	**Welfare**	**M**	**NM**	**Employment-M**	**Import**
U.S.	-4.12%	-3.20%	0.44%	-3.29%	-19.40%	-5.22%	-0.56%	0.08%	-0.57%	-29.00%
China	-5.64%	-2.01%	0.84%	-2.23%	-28.35%	-6.42%	-3.84%	1.60%	-4.26%	-29.98%
World	-4.47%	-2.50%	0.56%	-2.68%	-23.61%	-5.50%	-2.50%	0.56%	-2.67%	-29.46%

	U.S. imposes 100% tariff, China imposes 50% tariff					**U.S. imposes 100% tariff, China imposes 100% tariff**				
Country	**Welfare**	**M**	**NM**	**Employment-M**	**Import**	**Welfare**	**M**	**NM**	**Employment-M**	**Import**
U.S.	-6.33%	-3.64%	0.50%	-3.73%	-31.24%	-8.14%	0.67%	-0.09%	0.69%	-43.62%
China	-8.79%	-2.90%	1.21%	-3.22%	-44.67%	-10.20%	-5.04%	2.10%	-5.57%	-46.89%
World	-6.91%	-3.20%	0.72%	-3.44%	-37.57%	-8.62%	-2.70%	0.61%	-2.88%	-45.16%

Table 5.4: 2 Country Simulation Results, $\sigma_i = 2 \quad \sigma_j = 0.7 \quad \sigma_l^q = \sigma_l^z = 2$

	U.S. Unilateral Tariff War (25%)					**U.S. China Mutual Tariff War (25%)**				
Country	**Welfare**	**M**	**NM**	**Employment-M**	**Import**	**Welfare**	**M**	**NM**	**Employment-M**	**Import**
U.S.	-1.31%	-4.98%	0.68%	-5.10%	-3.62%	-2.84%	-3.67%	0.50%	-3.77%	-17.76%
China	-2.24%	0.28%	-0.12%	0.31%	-15.99%	-3.93%	-2.61%	1.09%	-2.90%	-19.14%
World	-1.53%	-1.87%	0.42%	-2.02%	-9.45%	-3.10%	-3.05%	0.69%	-3.27%	-18.41%

	U.S. imposes 50% tariff, China imposes 25% tariff					**U.S. imposes 50% tariff, China imposes 50% tariff**				
Country	**Welfare**	**M**	**NM**	**Employment-M**	**Import**	**Welfare**	**M**	**NM**	**Employment-M**	**Import**
U.S.	-3.69%	-6.80%	0.93%	-6.97%	-19.89%	-4.82%	-5.48%	0.75%	-5.62%	-29.55%
China	-5.58%	-2.19%	0.91%	-2.44%	-30.19%	-6.74%	-4.02%	1.68%	-4.46%	-32.37%
World	-4.13%	-4.08%	0.92%	-4.39%	-24.74%	-5.27%	-4.62%	1.04%	-4.96%	-30.88%

	U.S. imposes 100% tariff, China imposes 50% tariff					**U.S. imposes 100% tariff, China imposes 100% tariff**				
Country	**Welfare**	**M**	**NM**	**Employment-M**	**Import**	**Welfare**	**M**	**NM**	**Employment-M**	**Import**
U.S.	-5.83%	-9.23%	1.25%	-9.45%	-31.77%	-7.42%	-6.71%	0.91%	-6.87%	-44.12%
China	-9.11%	-3.14%	1.31%	-3.49%	-46.95%	-10.58%	-5.24%	2.18%	-5.80%	-49.66%
World	-6.60%	-5.63%	1.27%	-6.05%	-38.92%	-8.17%	-5.84%	1.32%	-6.26%	-46.73%

Chapter 6

Conclusion

This thesis presents the solution of each sector's optimization function, the Social Accounting Matrix database and the simulation results of the relative change of welfare, production, employment and the trade volume due to U.S.-China trade war under a computable general equilibrium model. Both the U.S. and China suffer the loss of welfare, which will be more significant when China retaliates. Though U.S. is expected to gain in manufacturing sector under tariff retaliation scenario, the negative global effect is attributed to the larger negative magnitudes in China's manufacturing. Also, from the sensitivity analysis, the relative change is subtle when the elasticity of technology transformation ranges between 0.7 to 1.25. When the elasticity of substitution between manufactured goods and non-manufactured goods is between 0.5 to 2, higher value of elasticity reduces both countries' welfare loss but enlarge the damage to the U.S. manufacturing production.

Simulation results in this paper are mostly consistent with other literatures with the Armington assumption at the aggregate level. Higher tariff induces higher trade loss and and welfare loss for both the U.S. and China. The world is also hurt in all indicators except the non-manufacturing sector. Also, retaliation from China clearly induces more damage than the non-retaliation trade policy. The only difference in the result is the manufacturing sector. Some literatures point out that the U.S.'s manufacturing would be better off if China doesn't retaliate, but the country's manufacturing would be worse off if China retaliates. In contrast, in this paper when the consumption demand elasticity is set as 0.5, the result presents that the U.S. manufacturing loses when the U.S. imposes a higher tariff than China, but the loss reduces with the rise of tariffs. Rather than losing when China retaliates, in a bilateral trade model, the U.S. gains when China imposes same percentage of tariff. However, the gain in the U.S. manufacturing disappears when the demand elasticity is above 1, which means higher substitutability. The higher tariffs, and the easiness to substitute may cause the non-manufactured goods more preferred and decrease the non-manufacturing production.It's also interesting to see that when the U.S. imposes a unilateral tariff on Chinese imports, the U.S. is the one who suffers from loss of exports in the long run. It might be the reason that lower demand of Chinese goods decreases the price, then in the long run, producers prefer Chinese intermediate goods for production, and consumers prefer Chinese goods for consumption other than the American goods.

As a bilateral trade model, the standard CGE model used in the simulation in 2-country has several limitation. Firstly, if a country imposes tariff, the tariff is implemented on every other country in the world, and a mutual trade war between the U.S. and China is equivalent to a tariff war that includes all countries in the world.

This could be the main reason that the relative change of welfare is more than 1%, arriving at a different result compared to Balistreri (2018) [4] that the change of welfare is usually within 1% adopting the Armington, constant-return-to-scale framework general equilibrium model. When there are only two countries in the world, trade diversion is ambiguous since the trade flow has nowhere to diversify other than the country who imposes the higher tariff.

Secondly, when constructing the database, some pairs of trade data have to be adjusted to fit in the general equilibrium assumption. In other words, the value of export plus foreign saving must be equal to the value of import, which is also the value of the other country's export. Violating this condition will induce an error after running the simulation because the number in the database must be the general equilibrium outcome from the previous period by assumption.

It's also worth noting that one of the disadvantages of static model is that some key indicators such as investment and the income from saving are not as important as they are in a dynamic long-run model that captures the results of several short-run periods. In the static model, one-time solution omits the path of convergence to equilibrium. However, the biggest advantage of static aggregated CGE model is also obvious that the static Armington type of model indicates how trade policy influences at the country's level, and how should the policymaker react for a better final result in this period. And th fixed total input factor assumption automatically benefits the non-manufacturing sector when manufacturing sector suffers from a loss, and vice-versa. No wonder the sign of relative change of the two sectors' production is inverse.

Overall, there's a lot of room for this simulation to improve: using a dynamic model to simulate instead of a static model; rewriting the model that includes a tariff function that only targeting at a particular country and including more countries, industries or types of input factors in the database. Though most international trade policy reform simulation is based on the Armington type of CGE model which facilitates trade aggregation, the Armington assumption has some loopholes. Balistreri and Markusen (2009)[3] criticizes that the Armington type of model has implicit market power by heterogeneity in a perfectly competitive market, and it mis-allocates market power over varieties away from firms and toward the discretion of the policy authority. As Balistreri(2018)[4] simulates the trade war scenario by Armington, Krugman and Bilateral Representative Firms type of models respectively, and it's also worth to compare the simulation results by different model structures in the future.

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Appendix A

Nested-utility Expression

The optimization procedure of nested utility function under Armington assumption in (Li et al., 2018)[25] is shown below. From the utility optimization, each country chooses the optimal bundle of domestic and foreign goods, and the exact value from each foreign country. The composite foreign good price is also determined. Here's the list of notations used in equations (A.1) to (A.34).

i = The country of consumption j = The country of origin

T = Tradable good NT = Non-tradable good

M_i = money supply of country i (fixed)

X_i^{NT} = Consumption of non-tradable (domestic) goods in country i

X_i^T = Consumption of Composite tradable goods in country i x_{ij}^T = Comsumption of good in country i produced in country j

P_i^T = Price of composite tradable goods in country i

pc_{ij}^T = Price of the tradable goods originated from country j pc_i^{NT} = Price of domestic goods in country i

σ_i = Armington Elasticity between tradable goods and non-tradable goods

$\sigma_i^{'}$ = Armington Elasticity beteween countries

$$U_i(X_i^T, X_i^{NT}) = [\alpha_{i1}^{\frac{1}{\sigma_i}}(X_i^T)^{\frac{\sigma_{i-1}}{\sigma_i}} + \alpha_{i2}^{\frac{1}{\sigma_i}}(X_i^{NT})^{\frac{\sigma_{i-1}}{\sigma_i}}]^{\frac{\sigma_i}{\sigma_{i-1}}} \tag{A.1}$$

$$s.t. P_i^T X_i^T + pc_i^{NT} X_i^{NT} = \bar{M}_i$$

For a constraint optimization problem, a Lagrangian needs to be built.

$$\mathcal{L}(X_i^T, X_i^{NT}) = U_i(X_i^T, X_i^{NT}) + \lambda(\bar{M}_i - P_i^T X_i^T - pc_i^{NT} X_i^{NT}) \tag{A.2}$$

Take the partial derivative with respect to X_i^T,X_i^{NT} and λ, then compute the first order condition.

$$\frac{\partial \mathcal{L}}{\partial X_i^T} = \frac{\partial U_i}{\partial X_i^T} - \lambda P_i^T = 0 \tag{A.3}$$

$$\frac{\partial \mathcal{L}}{\partial X_i^{NT}} = \frac{\partial U_i}{\partial X_i^{NT}} - \lambda pc_i^{NT} = 0 \tag{A.4}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \bar{M}_i - P_i^T X_i^T - pc_i^{NT} X_i^{NT} = 0 \tag{A.5}$$

Rearrange function (A.3) and (A.4)

$$\alpha_{i1}^{\frac{1}{\sigma_i}} \frac{\sigma_{i-1}}{\sigma_i} (X_i^T)^{\frac{-1}{\sigma_i}} = \lambda P_i^T \tag{A.6}$$

$$\alpha_{i2}^{\frac{1}{\sigma_i}} \frac{\sigma_{i-1}}{\sigma_i} (X_i^{NT})^{\frac{-1}{\sigma_i}} = \lambda pc_i^{NT} \tag{A.7}$$

Take the ratio of function (A.6) and (A.7)

$$\frac{P_i^T}{pc_i^{NT}} = \frac{\alpha_{i1}^{\frac{1}{\sigma_i}} \frac{\sigma_{i-1}}{\sigma_i} (X_i^T)^{\frac{-1}{\sigma_i}}}{\alpha_{i2}^{\frac{1}{\sigma_i}} \frac{\sigma_{i-1}}{\sigma_i} (X_i^{NT})^{\frac{-1}{\sigma_i}}}$$

$$(\frac{P_i^T}{pc_i^{NT}})^{\sigma_i} = \frac{\alpha_{i1} X_i^{NT}}{\alpha_{i2} X_i^T} \tag{A.8}$$

$$X_i^{NT} = \frac{\alpha_{i2}}{\alpha_{i1}}(\frac{P_i^T}{pc_i^{NT}})^{\sigma_i} X_i^T \tag{A.9}$$

$$P_i^T X_i^T + pc_i^{NT}\frac{\alpha_{i2}}{\alpha_{i1}}(\frac{P_i^T}{pc_i^{NT}})^{\sigma_i} X_i^T = \bar{M}_i \tag{A.10}$$

$$X_i^T = \bar{M}_i \frac{1}{P_i^T + pc_i^{NT}\frac{\alpha_{i2}}{\alpha_{i1}}(\frac{P_i^T}{pc_i^{NT}})^{\sigma_i}} = \frac{\alpha_{i1}\bar{M}_i}{(P_i^T)^{\sigma_i}[\alpha_{i1}(P_i^T)^{1-\sigma_i} + \alpha_{i2}(pc_i^{NT})^{1-\sigma_i}]} \tag{A.11}$$

Plug the result in (A.10) which yields X_i^{NT}

$$X_i^{NT} = \frac{\alpha_{i2}}{\alpha_{i1}}(\frac{P_i^T}{pc_i^{NT}})^{\sigma_i} \frac{\alpha_{i1}\bar{M}_i}{(P_i^T)^{\sigma_i}[\alpha_{i1}(P_i^T)^{1-\sigma_i} + \alpha_{i2}(pc_i^{NT})^{1-\sigma_i}]} \tag{A.12}$$

$$X_i^{NT} = \frac{\alpha_{i2}\bar{M}_i}{(pc_i^{NT})^{\sigma_i}[\alpha_{i1}(P_i^T)^{1-\sigma_i} + \alpha_{i2}(pc_i^{NT})^{1-\sigma_i}]} \tag{A.13}$$

Equations (A.11) and (A.13) represents the optimal X_i^{NT} and X_i^T maximizing the first level preference $U_i(X_i^T, X_i^{NT})$ which is determined by σ_i. For a specific country, it's aiming at allocating the proportion of different tradable goods from each country j, $j = 1, 2, ...m$ to maximize their second level of preference.

$$X_i^T = [\sum_{j=1}^{\infty} \beta_{ij}^{\frac{1}{\sigma_i'}} x_{ij}^{T\frac{\sigma_i'-1}{\sigma_i'}}]^{\frac{\sigma_i'}{\sigma_i'-1}} \tag{A.14}$$

To solve for x_{ij}^T, a Lagragian is set up to maximize X_i^T.

$$\mathcal{L}(x_{ij}) = [\sum_{j=1}^{\infty} \beta_{ij}^{\frac{1}{\sigma_i'}} x_{ij}^{T\frac{\sigma_i'-1}{\sigma_i'}}]^{\frac{\sigma_i'}{\sigma_i'-1}} + \lambda(X_i^T P_i^T - \sum_{j=1}^{\infty} x_{ij}^T pc_{ij}^T) \tag{A.15}$$

Since this function is symmetric, x_{i1} is solved to represent all other x_{ij}. For the first order condition, take the partial derivative of x_{i1}

$$\frac{\partial \mathcal{L}}{\partial x_{i1}^T} = \frac{\sigma_i^{'}}{\sigma_i^{'}-1}[\sum_{j=1}^{\infty} \beta_{ij}^{\frac{1}{\sigma_i^{'}}} x_{ij}^{T\frac{\sigma_i^{'}-1}{\sigma_i^{'}}}]^{\frac{1}{\sigma_i^{'}-1}} \beta_{i1}^{\frac{1}{\sigma_i^{'}}} \frac{\sigma_i^{'}}{\sigma_i^{'}-1} x_{i1}^{T\frac{-1}{\sigma_i}} - \lambda pc_{i1}^T = 0 \tag{A.16}$$

For the general terms x_{ij}^T

$$\frac{\partial \mathcal{L}}{\partial x_{ij}^T} = \frac{\sigma_i^{'}}{\sigma_i^{'}-1}[\sum_{j=1}^{\infty} \beta_{ij}^{\frac{1}{\sigma_i^{'}}} x_{ij}^{T\frac{\sigma_i^{'}-1}{\sigma_i^{'}}}]^{\frac{1}{\sigma_i^{'}-1}} \beta_{ij}^{\frac{1}{\sigma_i^{'}}} \frac{\sigma_i^{'}}{\sigma_i^{'}-1} x_{ij}^{T\frac{-1}{\sigma_i}} - \lambda pc_{ij}^T = 0 \tag{A.17}$$

$$\frac{\sigma_i^{'}}{\sigma_i^{'}-1}[\sum_{j=1}^{\infty} \beta_{ij}^{\frac{1}{\sigma_i^{'}}} x_{ij}^{T\frac{\sigma_i^{'}-1}{\sigma_i^{'}}}]^{\frac{1}{\sigma_i^{'}-1}} \beta_{i1}^{\frac{1}{\sigma_i^{'}}} \frac{\sigma_i^{'}}{\sigma_i^{'}-1} x_{i1}^{T\frac{-1}{\sigma_i}} = \lambda pc_{i1}^T \tag{A.18}$$

$$\frac{\sigma_i^{'}}{\sigma_i^{'}-1}[\sum_{j=1}^{\infty} \beta_{ij}^{\frac{1}{\sigma_i^{'}}} x_{ij}^{T\frac{\sigma_i^{'}-1}{\sigma_i^{'}}}]^{\frac{1}{\sigma_i^{'}-1}} \beta_{ij}^{\frac{1}{\sigma_i^{'}}} \frac{\sigma_i^{'}}{\sigma_i^{'}-1} x_{ij}^{T\frac{-1}{\sigma_i}} = \lambda pc_{ij}^T \tag{A.19}$$

Take the ratio of λp_{i1}^T and λp_{ij}^T

$$\frac{\lambda pc_{i1}^T}{\lambda pc_{ij}^T} = \frac{\beta_{i1}^{\frac{1}{\sigma_i^{'}}} x_{i1}^{T\frac{-1}{\sigma_i}}}{\beta_{ij}^{\frac{1}{\sigma_i^{'}}} x_{ij}^{T\frac{-1}{\sigma_i}}} = (\frac{\beta_{i1} x_{ij}^T}{\beta_{ij} x_{i1}^T})^{\frac{1}{\sigma_i^{'}}} \tag{A.20}$$

$$(\frac{pc_{i1}^T}{pc_{ij}^T})^{\sigma_i^{'}} = \frac{\beta_{i1} x_{ij}^T}{\beta_{ij} x_{i1}^T} \tag{A.21}$$

$$x_{ij}^T = (\frac{pc_{i1}^T}{pc_{ij}^T})^{\sigma_i^{'}} \frac{\beta_{ij}}{\beta_{i1}} x_{i1}^T \tag{A.22}$$

Use the expression of x_{ij}^T to calculate every term of $pc_{ij}^T x_{ij}^T$

$$pc_{i1}^T x_{i1}^T = (\frac{pc_{i1}^T}{pc_{i1}^T})^{\sigma_i^{'}} \frac{\beta_{i1}}{\beta_{i1}} x_{i1}^T pc_{i1}^T \tag{A.23}$$

$$pc_{i2}^{T}x_{i2}^{T} = (\frac{pc_{i1}^{T}}{pc_{i2}^{T}})^{\sigma_i'}\frac{\beta_{i2}}{\beta_{i1}}x_{i1}^{T}pc_{i2}^{T} = x_{i1}^{T}\frac{(pc_{i1}^{T})^{\sigma_i'}}{\beta_{i1}}\frac{\beta_{i2}}{(pc_{i2}^{nT})^{\sigma_i'}} \tag{A.24}$$

$$pc_{im}^{NT}x_{im}^{T} = (\frac{pc_{i1}^{T}}{pc_{in}^{T}})^{\sigma_i'}\frac{\beta_{im}}{\beta_{i1}}x_{i1}^{T}pc_{im}^{T} = x_{i1}^{T}\frac{(pc_{i1}^{T})^{\sigma_i'}}{\beta_{i1}}\frac{\beta_{im}}{(pc_{im}^{T})^{\sigma_i'}} \tag{A.25}$$

sum all the terms from (A.23) to (A.25)

$$\sum_{j=1}^{\infty} x_{ij}^{T}pc_{ij}^{T} = \frac{x_{i1}^{T}}{\beta_{i1}}(pc_{i1}^{T})^{\sigma_i'}\sum_{j=1}^{\infty}(\frac{\beta_{ij}}{(pc_{ij}^{T})^{\sigma_i'-1}}) = X_i^T P_i^T \tag{A.26}$$

$$x_{i1}^{T} = \frac{\beta_{i1}X_i^T P_i^T}{(pc_{i1}^{T})^{\sigma_i'}\sum_{j=1}^{\infty}\beta_{ij}(pc_{ij}^{T})^{1-\sigma_i'}} \tag{A.27}$$

since every term of x_{ij}^{T} is symmetric,

$$x_{ij}^{T} = \frac{\beta_{ij}X_i^T P_i^T}{(pc_{ij}^{T})^{\sigma_i'}\sum_{j=1}^{\infty}\beta_{ij}(pc_{ij}^{T})^{1-\sigma_i'}} \tag{A.28}$$

By the steps above, the second level preference is solved showing countries allocate their trade from other different countries. With the knowledge of x_{ij}^{T}, consumption price of tradable goods for each country can be calculated. plug x_{ij}^{T} into the constraint of equation (A.14) in the condition of optimization.

$$\sum_{j=1}^{\infty} x_{ij}^{T}pc_{ij}^{T} = X_i^T P_i^T \tag{A.29}$$

$$\sum_{j=1}^{\infty}[(\frac{\beta_{ij}X_i^T P_i^T}{(pc_{ij}^{T})^{\sigma_i'}\sum_{j=1}^{\infty}\beta_{ij}(pc_{ij}^{T})^{1-\sigma_i'}})pc_{ij}^{T}] = X_i^T P_i^T \tag{A.30}$$

Factor out the common term, the left hand equals to

$$\frac{X_i^T P_i^T}{\sum_{j=1}^{\infty} \beta_{ij} (pc_{ij}^T)^{1-\sigma_i^{'}}} [\frac{\beta_{i1} pc_{i1}^T}{(pc_{i1}^T)^{\sigma_i^{'}}} + \frac{\beta_{i2} pc_{i2}^T}{(pc_{i2}^T)^{\sigma_i^{'}}} + ... + \frac{\beta_{in} pc_{in}^T}{(pc_{in}^T)^{\sigma_i^{'}}}] = X_i^T P_i^T \tag{A.31}$$

Rearrange the equation

$$\beta_{i1} (pc_{i1}^T)^{1-\sigma_i^{'}} + \beta_{i2} (pc_{i2}^T)^{1-\sigma_i^{'}} + ... + \beta_{in} (pc_{in}^T)^{1-\sigma_i^{'}} = \sum_{j=1}^{\infty} \beta_{ij} (pc_{ij}^T)^{1-\sigma_i^{'}} \tag{A.32}$$

For the reason that the left hand side equals to $\sum_{j=1}^{\infty} \beta_{ij} (pc_{ij}^T)^{1-\sigma_i^{'}}$ and by Von Neumann-Morgenstern it equals to $(P_i^T)^{1-\sigma_i^{'}}$.Thus,

$$(P_i^T)^{1-\sigma_i^{'}} = \sum_{j=1}^{\infty} \beta_{ij} (pc_{ij}^T)^{1-\sigma_i^{'}} \tag{A.33}$$

$$P_i^T = (\sum_{j=1}^{\infty} \beta_{ij} (pc_{ij}^T)^{1-\sigma_i^{'}})^{\frac{1}{1-\sigma_i^{'}}} \tag{A.34}$$

Appendix B

GAMS Code

```
CONOPT 3         26.1.0 rf2b37b9 Released Feb 02, 2019 DEG x86 64bit/Mac OS X


C O N O P T 3   version 3.17I
Copyright (C)   ARKI Consulting and Development A/S
Bagsvaerdvej 246 A
DK-2880 Bagsvaerd, Denmark


Iter Phase Ninf   Infeasibility   RGmax    NSB   Step InItr MX OK
0   0        8.8798950904E+01 (Input point)

Pre-triangular equations:   2
Post-triangular equations:  3
Definitional equations:     28

1   0        1.1508299402E+02 (After pre-processing)
2   0        9.4983893976E+00 (After scaling)
10   0     2  1.0746085409E-02               1.0E+00      F  T
20   0     2  7.4864672751E-06               1.0E+00      F  T

```

```


Set
   u    'SAM entry' / MAN, NMA, CAP, LAB, IDT, TRF, HOH, GOV, INV, EXT /
   l(u) 'goods'     / MAN, NMA       /
   s(u) 'input factor'    /  CAP,LAB    /
   i    'country'   / CHN,USA  /;

Alias (u,v), (l,j), (s,ss), (i,r);

Table SAM(u,v,i) 'social accounting matrix'
         MAN.CHN NMA.CHN CAP.CHN LAB.CHN IDT.CHN
   MAN        78      32
   NMA        39      44
   CAP        17      29
   LAB        13      43
   IDT      4.55    5.05
   TRF      0.82    0.15
   HOH                        46      56
   GOV                                       9.6
   INV
   EXT        15       9

   +     TRF.CHN HOH.CHN GOV.CHN INV.CHN EXT.CHN
   MAN               5.6       2   24.22   25.55
   NMA                32      11    34.8     1.4
   CAP
   LAB
   IDT
   TRF
   HOH
   GOV      0.97      28
   INV              36.4   25.57           -2.95
   EXT

   +     MAN.USA NMA.USA CAP.USA LAB.USA IDT.USA
   MAN        16      15
```

```
    NMA        18      73
    CAP        11      65
    LAB       9.8      88
    IDT         0       0
    TRF      0.38    0.12
    HOH                        76    97.8
    GOV                                          0
    INV
    EXT     25.55     1.4

    +     TRF.USA HOH.USA GOV.USA INV.USA EXT.USA
    MAN                18       4   12.73      15
    NMA               104      22   16.52       9
    CAP
    LAB
    IDT
    TRF
    HOH
    GOV       0.5   41.77
    INV             10.03   16.27            2.95
    EXT                           ;
* loading the initial values


Parameter
Y0(j,i) 'aggregate output j  for eachcountry i'
F0(s,j,i) 'the s-th input facotr input by the j-th firm in country i'
X0(l,j,i) 'the intermediate inout from l-th frim to the j-th firm in country i'
Z0(j,i) 'output of the j-th good in country i'
Xh0(l,i) 'household consumption of the l-th industry good'
Xg0(l,i) 'government consumption of the l-th industry good'
Xv0(l,i) 'investment demand'
E0(l,i) 'exports'
M0(l,i) 'imports'
Q0(l,i) 'Armington composite good of imports and domestic goods for consumption'
D0(l,i) 'domestic good'
Sp0(i) 'private saving'
```

```
Sg0(i) ‘government saving’
Td0(i) ‘direct tax’
Tp0(j,i) ‘production tax’
Tm0(j,i) ‘import tariff’
FF(s,i) ‘factor endowment of the s-facotr in country i’
Sf(i) ‘foreign saving’
taxindirect (l,i) ‘indrect tax rate’
tariff_rate(l,i) ‘import tariff’;


Td0(i)    =  SAM("GOV","HOH",i);
Tp0(j,i)  =  SAM("IDT",j,i);
Tm0(j,i)  =  SAM("TRF",J,i);
F0(s,j,i) =  SAM(s,j,i);
Y0(j,i)   =  sum(s, F0(s,j,i));
X0(l,j,i) =  SAM(l,j,i);
Z0(j,i)   =  Y0(j,i) + sum(l, X0(l,j,i));
M0(l,i)   =  SAM("EXT",l,i);
taxindirect (j,i) =  Tp0(j,i)/Z0(j,i);
tariff_rate(j,i) =  Tm0(j,i)/M0(j,i);
Xh0(l,i)  =  SAM(l,"HOH",i);
FF(s,i)   =  SAM("HOH",s,i);
Xg0(l,i)  =  SAM(l,"GOV",i);
Xv0(l,i)  =  SAM(l,"INV",i);
E0(l,i)   =  SAM(l,"EXT",i);
Q0(l,i)   = (Xh0(l,i)+Xg0(l,i)+Xv0(l,i) + sum(j, X0(l,j,i)));
D0(l,i)   = (1+taxindirect (l,i))*Z0(l,i)-E0(l,i);
Sp0(i)    =  SAM("INV","HOH",i);
Sg0(i)    =  SAM("INV","GOV",i);
Sf(i)     =  SAM("INV","EXT",i);

display Y0, F0, Z0,   X0, Xh0, Xg0, Xv0, E0, M0, Q0, D0, Sp0, Sg0, Td0, Tp0, Tm0
        FF, Sf, taxindirect , tariff_rate;

* calibration
Parameter
  sigma1(i)  ‘elasticity of substitution’
```

```
  sigma2(j,i) ‘elasticity of factor input techonology transformation ’
  sigma3(l) ‘elasticity of substitution between imports and domestics consumption’
  sigma4(l)   ‘elasticity of transformation domestic and exports production’


   ro1(i) ‘elasticity parameter 1 country utility’
   ro2(j,i) ‘elasticity parameter 2 factor input’
   ro3(l)   ‘substitution elasticity parameter imports and domestics’
   ro4(l)   ‘transformation elasticity parameter domestic and exports’;

sigma1(i) = 2;
sigma2(j,i)  = 2;
sigma3(l) =  2;
sigma4(l)   =  2;



ro1(i)    = (sigma1(i)-1)/sigma1(i);
ro2(j,i) = (sigma2(j,i)-1)/sigma2(j,i);
ro3(l)   = (sigma3(l)-1)/sigma3(l);
ro4(l)   = (sigma4(l)+1)/sigma4(l);


Parameter
   alpha(l,i)  ‘share parameter in utility function’
   delta(s,j,i) ‘share parameter in production function’
   phi(j,i)      ‘scale parameter in production function’
   ax(l,j,i)   ‘coefficient for minimum requirements of the i-th intermediate inputs for one unit of gross outputs’
   ay(j,i)     ‘coefficient for minimum requirements of value added for one unit of gross outputs’
   mu(l,i)     ‘government consumption share’
   lambda(l,i) ‘investment demand share’
   alpham(l,i) ‘share parameter in Armington function’
   alphad(l,i) ‘share parameter in Armington function’
   gamma(l,i)  ‘scale parameter in Armington function’
   xid(l,i)    ‘share parameter in transformation function’
```

```
   xie(l,i)    ‘share parameter in transformation function’
   theta(l,i)  ‘scale parameter in transformation function’
   ssp(i)      ‘propensity for private saving’
   ssg(i)      ‘propensity for government saving’
   direct_tax(i)     ‘direct tax rate’;

alpha(l,i)  =    Xh0(l,i)/sum(j, Xh0(j,i));
delta(s,j,i) =  F0(s,j,i)**(1-ro2(j,i)) /sum(ss, F0(ss,j,i)**(1-ro2(j,i)));
phi(j,i)      =  Y0(j,i)/(sum(s,delta(s,j,i)*F0(s,j,i)**ro2(j,i))**(1/ro2(j,i)));
ax(l,j,i)   =  X0(l,j,i)/Z0(j,i);
ay(j,i)     =  Y0(j,i)/Z0(j,i);
mu(l,i)     =  Xg0(l,i)/sum(j, Xg0(j,i));
lambda(l,i) =  Xv0(l,i)/(Sp0(i)+Sg0(i)+Sf(i));
alpham(l,i) = (1+tariff_rate(l,i))*M0(l,i)**(1-ro3(l))/((1+tariff_rate(l,i))*M0(l,
    i)**(1-ro3(l)) + D0(l,i)**(1-ro3(l)));
alphad(l,i) =  D0(l,i)**(1-ro3(l))/((1+tariff_rate(l,i))*M0(l,i)**(1-ro3(l)) +D0(l
    ,i)**(1-ro3(l)));
gamma(l,i)  =  Q0(l,i)/(alpham(l,i)*M0(l,i)**ro3(l) + alphad(l,i)*D0(l,i)**ro3(l))
    **(1/ro3(l));
xie(l,i)    =  E0(l,i)**(1-ro4(l))/(E0(l,i)**(1-ro4(l))+D0(l,i)**(1-ro4(l)));
xid(l,i)    =  D0(l,i)**(1-ro4(l))/(E0(l,i)**(1-ro4(l))+D0(l,i)**(1-ro4(l)));
theta(l,i)  =  Z0(l,i)/(xie(l,i)*E0(l,i)**ro4(l) + xid(l,i)*D0(l,i)**ro4(l))**(1/
    ro4(l));
ssp(i)      =  Sp0(i)/sum(s, FF(s,i));
ssg(i)      =  Sg0(i)/(Td0(i)+sum(j, Tp0(j,i))+sum(j, Tm0(j,i)));
direct_tax(i)     =  Td0(i)/sum(s, FF(s,i));

display alpha, delta,  phi,   ax,  ay, mu, lambda, alpham, alphad, gamma, xie
        xid,   theta, ssp, ssg, direct_tax;

Variable
   Y(j,i)     ‘composite factor’
   F(s,j,i)   ‘the s-th factor input by the j-th industry’
   X(l,j,i)   ‘intermediate input l-th industry to j-th industry’
   Z(j,i)     ‘output of the j-th good’
   Xh(l,i)    ‘household cons. of the i-th good’
   Xg(l,i)    ‘government consumption’
```

```
   Xv(l,i)    ‘investment demand’
   E(l,i)     ‘exports’
   M(l,i)     ‘imports’
   Q(l,i)     ‘Armington’s composite good’
   D(l,i)     ‘domestic good’
   pf(s,i)    ‘the s-th input factor price’
   py(j,i)    ‘composite factor price’
   pz(l,i)    ‘supply price of the l-th good’
   pq(l,i)    ‘Armington’s composite good price’
   pe(l,i)    ‘export price in local currency’
   pm(l,i)    ‘import price in local currency’
   pd(l,i)    ‘the l-th domestic good price’
   epsilon(i) ‘exchange rate’
   pWe(l,i)   ‘export price in US dollars’
   pWm(l,i)   ‘import price in US dollars’
   Sp(i)      ‘private saving’
   Sg(i)      ‘government saving’
   Td(i)      ‘direct tax’
   Tp(j,i)    ‘production tax’
   Tm(l,i)    ‘import tariff’
   A(i)       ‘the supporting sum’
   UU(i)      ‘utility’
   SW         ‘social welfare [fictitious obj. function]’;

Equation
   eqpy(j,i)    ‘composite factor aggregation function.’
   eqX(l,j,i)   ‘intermediate demand function’
   eqY(j,i)     ‘composite factor demand function.’
   eqF(s,j,i)   ‘factor demand function’
   eqpzs(j,i)   ‘unit cost function’
   eqTd(i)      ‘direct tax revenue function’
   eqTp(j,i)    ‘production tax revenue function’
   eqTm(l,i)    ‘import tariff revenue function’
   eqXg(l,i)    ‘government demand function’
   eqXv(l,i)    ‘investment demand function’
   eqSp(i)      ‘private saving function’
   eqSg(i)      ‘government saving function’
```

```
   eqXh(l,i)    ‘household demand function’
   eqpe(l,i)    ‘world export price equation’
   eqpm(l,i)    ‘world import price equation’
   eqepsilon(i) ‘balance of payments’
   eqpqs(l,i)   ‘Armington function’
   eqM(l,i)     ‘import demand function’
   eqD(l,i)     ‘domestic good demand function’
   eqpzd(l,i)   ‘transformation function’
   eqDs(l,i)    ‘domestic good supply function’
   eqE(l,i)     ‘export supply function’
   eqpw(l,i,r) ‘international price equilibrium’
   eqw(l,i,r)  ‘international quantity equilibrium’
   eqpqd(l,i)   ‘market clearing cond. for comp. good’
   eqpf(s,i)    ‘factor market clearing condition’
   eqaddition(i) ‘supporting function’
   eqUU(i)      ‘utility function’
   obj          ‘social welfare function [fictitious]’;

* domestic production
eqpy(j,i)..      Y(j,i)  =e= phi(j,i)*sum(s,delta(s,j,i)*F(s,j,i)**ro2(j,i))**(1/
    ro2(j,i));

eqX(l,j,i)..    X(l,j,i)=e= ax(l,j,i)*Z(j,i);

eqY(j,i)..      Y(j,i)  =e= ay(j,i)*Z(j,i);

eqF(s,j,i)..    F(s,j,i)=e= (((phi(j,i)**ro2(j,i))*py(j,i)*delta(s,j,i) /pf(s,i))
    **(1/(1-ro2(j,i))))*Y(j,i);

eqpzs(j,i)..    pz(j,i) =e= ay(j,i)*py(j,i) + sum(l, ax(l,j,i)*pq(l,i));

* government behavior
eqTd(i)..       Td(i)   =e= direct_tax(i)*sum(s, pf(s,i)*FF(s,i));

eqTp(l,i)..     Tp(l,i) =e= taxindirect (l,i)*pz(l,i)*Z(l,i);

eqTm(l,i)..     Tm(l,i) =e= tariff_rate(l,i)*pm(l,i)*M(l,i);
```

```

eqXg(l,i)..     Xg(l,i) =e= mu(l,i)*(Td(i) + sum(j, Tp(j,i)) + sum(j, Tm(j,i))-Sg(i))/pq(l,i);

* investment behavior
eqXv(l,i)..     Xv(l,i) =e= lambda(l,i)*(Sp(i) + Sg(i) + epsilon(i)*Sf(i))/pq(l,i);

* savings
eqSp(i)..       Sp(i)   =e= ssp(i)*sum(s, pf(s,i)*FF(s,i));

eqSg(i)..       Sg(i)   =e= ssg(i)*(Td(i) + sum(j, Tp(j,i)) + sum(j, Tm(j,i)));

* household consumption
eqaddition(i).. A(i)=e=sum(l,alpha(l,i)*pq(l,i)**(1-sigma1(i)));

eqXh(l,i)..     Xh(l,i) =e= alpha(l,i)*(sum(s, pf(s,i)*FF(s,i)) -Sp(i) - Td(i))/(pq(l,i)**sigma1(i)*A(i));

* international trade
eqpe(l,i)..     pe(l,i) =e= epsilon(i)*pWe(l,i);

eqpm(l,i)..     pm(l,i) =e= epsilon(i)*pWm(l,i);

eqepsilon(i)..  sum(l, pWe(l,i)*E(l,i)) + Sf(i) =e= sum(l, pWm(l,i)*M(l,i));

* Armington function
eqpqs(l,i)..    Q(l,i)  =e= gamma(l,i)*(alpham(l,i)*M(l,i)**ro3(l) + alphad(l,i)*D(l,i)**ro3(l))**(1/ro3(l));

eqM(l,i)..      M(l,i)  =e= (gamma(l,i)**ro3(l)*alpham(l,i)*pq(l,i)/((1+tariff_rate(l,i))*pm(l,i)))**(1/(1-ro3(l)))*Q(l,i);

eqD(l,i)..      D(l,i)  =e= (gamma(l,i)**ro3(l)*alphad(l,i)*pq(l,i)/pd(l,i))**(1/(1-ro3(l)))*Q(l,i);

* transformation function
```

```
eqpzd(l,i)..    Z(l,i)  =e= theta(l,i)*(xie(l,i)*E(l,i)**ro4(l) + xid(l,i)*D(l,i)**ro4(l))**(1/ro4(l));
eqE(l,i)..      E(l,i)  =e= (theta(l,i)**ro4(l)*xie(l,i)*(1+taxindirect (l,i))*pz(l,i)/pe(l,i))**(1/(1-ro4(l)))*Z(l,i);
eqDs(l,i)..     D(l,i)  =e= (theta(l,i)**ro4(l)*xid(l,i)*(1+taxindirect (l,i))*pz(l,i)/pd(l,i))**(1/(1-ro4(l)))*Z(l,i);
* market clearing condition
eqpqd(l,i)..    Q(l,i)  =e= Xh(l,i)+Xg(l,i)+Xv(l,i) + sum(j, X(l,j,i));
eqpf(s,i)..     FF(s,i) =e= sum(j, F(s,j,i));
* international market clearing condition
eqpw(l,i,r)..  (pWe(l,i) -pWm(l,r))$(ord(i) <> ord(r)) =e= 0;
eqw(l,i,r)..   (E(l,i) -M(l,r))$(ord(i) <> ord(r)) =e= 0;
* fictitious objective function
eqUU(i)..       UU(i)   =e= sum(l,(alpha(l,i)**(1/sigma1(i)))*(Xh(l,i)**ro1(i)))**(1/ro1(i));
obj..           SW      =e= sum(i, UU(i));
* Initializing variables
Y.l(j,i)     = Y0(j,i);
F.l(s,j,i)   = F0(s,j,i);
X.l(l,j,i)   = X0(l,j,i);
Z.l(j,i)     = Z0(j,i);
Xh.l(l,i)    = Xh0(l,i);
Xg.l(l,i)    = Xg0(l,i);
Xv.l(l,i)    = Xv0(l,i);
Q.l(l,i)     = Q0(l,i);
E.l(l,i)     = E0(l,i);
M.l(l,i)     = M0(l,i);
D.l(l,i)     = D0(l,i);
```

```
pf.l(s,i)    = 1;
py.l(j,i)    = 1;
pz.l(l,i)    = 1;
pq.l(l,i)    = 1;
pe.l(l,i)    = 1;
pm.l(l,i)    = 1;
pd.l(l,i)    = 1;
A.l(i)       =1;
epsilon.l(i) = 1;
pWe.l(l,i)   = 1;
pWm.l(l,i)   = 1;
Sp.l(i)      = Sp0(i);
Sg.l(i)      = Sg0(i);
Td.l(i)      = Td0(i);
Tp.l(l,i)    = Tp0(l,i);
Tm.l(l,i)    = Tm0(l,i);

* setting lower bounds to avoid division by zero
Y.lo(j,i)      = 0.00001;
F.lo(s,j,i)    = 0.00001;
X.lo(l,j,i)    = 0.00001;
Z.lo(j,i)      = 0.00001;
Xh.lo(l,i)     = 0.00001;
Xg.lo(l,i)     = 0.00001;
Xv.lo(l,i)     = 0.00001;
Q.lo(l,i)      = 0.00001;
E.lo(l,i)      = 0.00001;
M.lo(l,i)      = 0.00001;
D.lo(l,i)      = 0.00001;
pf.lo(s,i)     = 0.00001;
py.lo(j,i)     = 0.00001;
pz.lo(l,i)     = 0.00001;
pq.lo(l,i)     = 0.00001;
pe.lo(l,i)     = 0.00001;
pm.lo(l,i)     = 0.00001;
pd.lo(l,i)     = 0.00001;
epsilon.lo(i) = 0.00001;
```

```
pWe.lo(l,i)   = 0.00001;
pWm.lo(l,i)   = 0.00001;
Sp.lo(i)      = 0.00001;
Sg.lo(i)      = 0.00001;
Td.lo(i)      = 0.00001;
Tp.lo(l,i)    = 0.0000;
Tm.lo(l,i)    = 0.0000;

* numeraire
pf.fx("LAB",i) = 1;

* fixing the redundant variable
epsilon.fx("USA") = 1;

Model twocge / all /;

solve twocge maximizing SW using nlp;

* Simulation Runs: 25% Import Tariffs from the U.S.
tariff_rate(j,"USA") = tariff_rate(j,"USA") + 0.25;

option bRatio = 1;

solve twocge maximizing SW using nlp;



* Display changes
Parameter
   dY(j,i), dF(s,j,i), dX(l,j,i), dZ(j,i) ,  dXh(l,i),    dXg(l,i), dXv(l,i)
  dQ(l,i),  dE(l,i),   dM(l,i),   dD(l,i),  dpf(s,i) ,    dpy(j,i) ,  dpz(l,i)
   dpq(l,i) ,dpe(l,i),  dpm(l,i),   dpd(l,i), dpWe(l,i), dpWm(l,i) , depsilon(i),
       dSp(i), dSg(i),dTd(i), dTp(l,i) , dTm(l,i);

dY(j,i)     = (Y.l(j,i)  /Y0(j,i)  -1)*100;
dF(s,j,i)   = (F.l(s,j,i)/F0(s,j,i)-1)*100;
dX(l,j,i)   = (X.l(l,j,i)/X0(l,j,i)-1)*100;
```

```
dZ(j,i)      = (Z.l(j,i)  /Z0(j,i)  -1)*100;
dXh(l,i)     = (Xh.l(l,i) /Xh0(l,i) -1)*100;
dXg(l,i)     = (Xg.l(l,i) /Xg0(l,i) -1)*100;
dXv(l,i)     = (Xv.l(l,i) /Xv0(l,i) -1)*100;
dQ(l,i)      = (Q.l(l,i)  /Q0(l,i)  -1)*100;
dE(l,i)      = (E.l(l,i)  /E0(l,i)  -1)*100;
dM(l,i)      = (M.l(l,i)  /M0(l,i)  -1)*100;
dD(l,i)      = (D.l(l,i)  /D0(l,i)  -1)*100;
dpf(s,i)     = (pf.l(s,i) /1 -1)*100;
dpy(j,i)     = (py.l(j,i) /1 -1)*100;
dpz(l,i)     = (pz.l(l,i) /1 -1)*100;
dpq(l,i)     = (pq.l(l,i) /1 -1)*100;
dpe(l,i)     = (pe.l(l,i) /1 -1)*100;
dpm(l,i)     = (pm.l(l,i) /1 -1)*100;
dpd(l,i)     = (pd.l(l,i) /1 -1)*100;
dpWe(l,i)    = (pWe.l(l,i)/1 -1)*100;
dpWm(l,i)    = (pWm.l(l,i)/1 -1)*100;
depsilon(i) = (epsilon.l(i)/1 -1)*100;
dSp(i)       = (Sp.l(i)   /Sp0(i)   -1)*100;
dSg(i)       = (Sg.l(i)   /Sg0(i)   -1)*100;
dTd(i)       = (Td.l(i)   /Td0(i)   -1)*100;
dTm(l,i)     = (Tm.l(l,i) /Tm0(l,i) -1)*100;

display dY,  dF,  dX,  dZ,  dXh, dXg,  dXv,  dQ,  dE,  dM,  dD,  dpf, dpy, dpz,
    dpq, dpe, dpm, dpd, dpWe, dpWm, depsilon, dTd,  dTm, dSp, dSg;
```

Appendix C

Results Using Other Elasticity Values

Table C.1: 2 Country Simulation, Cobb-Douglas Utility and Production Functions, $\sigma_i = \sigma_j \to 1, \quad \sigma_l^q = \sigma_l^z = 2$

	U.S. Unilateral Tariff War (25%)					**U.S. China Mutual Tariff War (25%)**				
Country	**Welfare**	**M**	**NM**	**Employment-M**	**Import**	**Welfare**	**M**	**NM**	**Employment-M**	**Import**
U.S.	-1.38%	-3.85%	0.52%	-3.95%	-3.41%	-3.04%	-0.70%	0.10%	-0.72%	-17.33%
China	-2.16%	0.38%	-0.16%	0.42%	-15.31%	-3.71%	-2.48%	1.03%	-2.75%	-17.46%
World	-1.56%	-1.35%	0.31%	-1.46%	-9.01%	-3.20%	-1.75%	0.40%	-1.88%	-17.39%

	U.S. imposes 50% tariff, China imposes 25% tariff					**U.S. imposes 50% tariff, China imposes 50% tariff**				
Country	**Welfare**	**M**	**NM**	**Employment-M**	**Import**	**Welfare**	**M**	**NM**	**Employment-M**	**Import**
U.S.	-4.12%	-3.20%	0.44%	-3.40%	-19.40%	-5.22%	-0.56%	0.08%	-0.57%	-29.01%
China	-5.64%	-2.01%	0.84%	-2.28%	-28.35%	-6.42%	-3.85%	1.60%	-4.26%	-29.98%
World	-4.47%	-2.50%	0.56%	-2.76%	-23.62%	-5.50%	-2.50%	0.56%	-2.68%	-29.46%

	U.S. imposes 100% tariff, China imposes 50% tariff					**U.S. imposes 100% tariff, China imposes 100% tariff**				
Country	**Welfare**	**M**	**NM**	**Employment-M**	**Import**	**Welfare**	**M**	**NM**	**Employment-M**	**Import**
U.S.	-6.33%	-3.64%	0.50%	-3.73%	-31.25%	-8.14%	0.68%	-0.09%	0.70%	-43.62%
China	-8.79%	-2.91%	1.21%	-3.22%	-44.67%	-10.19%	-5.04%	2.10%	-5.58%	-46.89%
World	-6.91%	-3.21%	0.72%	-3.44%	-37.57%	-8.62%	-2.70%	0.61%	-2.88%	-45.16%

Table C.2: 2 Country Simulation, $\sigma_i = 2, \quad \sigma_j \to 1, \quad \sigma_l^q = \sigma_l^z = 2$

	U.S. Unilateral Tariff War (25%)					**U.S. China Mutual Tariff War (25%)**				
Country	**Welfare**	**M**	**NM**	**Employment-M**	**Import**	**Welfare**	**M**	**NM**	**Employment-M**	**Import**
U.S.	-1.31%	-4.98%	0.68%	-5.11%	-3.63%	-2.84%	-3.68%	0.50%	-3.77%	-17.8%
China	-2.24%	0.28%	-0.12%	0.31%	-15.99%	-3.93%	-2.62%	1.09%	-2.90%	-19.1%
World	-1.53%	-1.88%	0.42%	-2.02%	-9.45%	-3.10%	-3.05%	0.69%	-3.28%	-18.4%

	U.S. imposes 50% tariff, China imposes 25% tariff					**U.S. imposes 50% tariff, China imposes 50% tariff**				
Country	**Welfare**	**M**	**NM**	**Employment-M**	**Import**	**Welfare**	**M**	**NM**	**Employment-M**	**Import**
U.S.	-3.69%	-6.81%	0.93%	-6.98%	-19.90%	-4.82%	-5.48%	0.75%	-5.62%	-29.56%
China	-5.58%	-2.20%	0.92%	-2.44%	-30.18%	-6.74%	-4.03%	1.68%	-4.47%	-32.37%
World	-4.13%	-4.09%	0.92%	-4.39%	-24.74%	-5.27%	-4.63%	1.04%	-4.96%	-30.88%

	U.S. imposes 100% tariff, China imposes 50% tariff					**U.S. imposes 100% tariff, China imposes 100% tariff**				
Country	**Welfare**	**M**	**NM**	**Employment-M**	**Import**	**Welfare**	**M**	**NM**	**Employment-M**	**Import**
U.S.	-5.83%	-9.23%	1.26%	-9.46%	-31.79%	-7.42%	-6.71%	0.91%	-6.87%	-44.13%
China	-9.10%	-3.15%	1.31%	-3.49%	-46.94%	-10.58%	-5.25%	2.19%	-5.81%	-49.66%
World	-6.60%	-5.64%	1.27%	-6.06%	-38.93%	-8.16%	-5.85%	1.32%	-6.27%	-46.74%

Table C.3: 2 Country Simulation, $\sigma_i \to 1, \quad \sigma_j = 2, \quad \sigma_l^q = \sigma_l^z = 2$

	U.S. Unilateral Tariff War (25%)					**U.S. China Mutual Tariff War (25%)**				
Country	**Welfare**	**M**	**NM**	**Employment-M**	**Import**	**Welfare**	**M**	**NM**	**Employment-M**	**Import**
U.S.	-1.38%	-3.85%	0.52%	-3.95%	-3.41%	-3.04%	-0.70%	0.10%	-0.72%	-17.34%
China	-2.16%	0.38%	-0.16%	0.42%	-15.30%	-3.71%	-2.49%	1.04%	-2.76%	-17.46%
World	-1.56%	-1.35%	0.31%	-1.46%	-9.01%	-3.20%	-1.76%	0.40%	-1.88%	-17.39%

	U.S. imposes 50% tariff, China imposes 25% tariff					**U.S. imposes 50% tariff, China imposes 50% tariff**				
Country	**Welfare**	**M**	**NM**	**Employment-M**	**Import**	**Welfare**	**M**	**NM**	**Employment-M**	**Import**
U.S.	-3.96%	-3.20%	0.44%	-3.28%	-19.41%	-5.22%	-3.85%	0.52%	-0.57%	-29.01%
China	-5.34%	-2.02%	0.84%	-2.24%	-28.35%	-6.42%	0.38%	-0.16%	-4.27%	-29.98%
World	-4.28%	-2.50%	0.56%	-2.69%	-23.62%	-5.50%	-2.50%	0.56%	-2.68%	-29.47%

	U.S. imposes 100% tariff, China imposes 50% tariff					**U.S. imposes 100% tariff, China imposes 100% tariff**				
Country	**Welfare**	**M**	**NM**	**Employment-M**	**Import**	**Welfare**	**M**	**NM**	**Employment-M**	**Import**
U.S.	-6.33%	-3.64%	0.49%	-3.73%	-31.26%	-8.14%	0.68%	-0.09%	0.70%	-43.63%
China	-8.79%	-2.91%	1.21%	-3.23%	-44.67%	-10.19%	-5.05%	-2.10%	-5.59%	-46.89%
World	-6.91%	-3.21%	0.72%	-3.45%	-37.58%	-8.62%	-2.70%	0.61%	-2.89%	-45.17%

Appendix D

When the Elasticity Parameter Converges to Special Values

Below I use production technology function to examine the functional form when the elasticity of technology substitution approaches to the values that has special properties. I use capital and labor, K and L for generalization, and demonstrate the functional variation when σ Approaches $\infty, 0$ and 1.

$$F(K,L) = [\gamma K^{\frac{\sigma-1}{\sigma}} + (1-\gamma)L^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}$$

The elasticity of substitution is $\epsilon_{KL} = \frac{dln(K/L)}{dln(F_K/F_L)}$

$$F_K = \gamma K^{\frac{-1}{\sigma}} F(K,L)^{\frac{1}{\sigma}}$$

$$F_L = (1-\gamma)L^{\frac{-1}{\sigma}} F(K,L)^{\frac{1}{\sigma}}$$

$$ln(\frac{F_K}{F_L}) = ln(\frac{\gamma}{1-\gamma}(\frac{K}{L})^{\frac{-1}{\sigma}}) = \frac{-1}{\sigma} ln((\frac{\gamma}{1-\gamma}(\frac{K}{L}))$$

$$\epsilon_{KL} = \frac{dln(K/L)}{dln(F_K/F_L)} = -\sigma\frac{1-\gamma}{\gamma}$$

From above, the elasticity of substitution is constant and doesn't depend on K or L.

If some extreme cases are observed when $\sigma \to \infty, \sigma \to 0, \sigma \to 1$

when $\sigma \to \infty$,$\lim_{\sigma\to\infty} \frac{\sigma-1}{\sigma} = 1$ then the production function is expressed as

$$\lim_{\sigma\to\infty} F(K,L) = \gamma K + (1-\gamma)L$$

Capital and labor are in the relation of perfect substitutes.

When $\sigma \to 0$, two conditions are needed to be considered. If $K \geq L$

$$\lim_{\sigma\to 0} \frac{F(K,L)}{L} = \lim_{\sigma\to 0}(\gamma(\frac{K}{L})^{\frac{\sigma-1}{\sigma}} + (1-\gamma))^{\frac{\sigma}{\sigma-1}}$$

Since $\frac{K}{L} > 1$ and $\lim_{\sigma\to 0} \frac{\sigma-1}{\sigma} \to -\infty$ so that $\lim_{\sigma\to 0}(\frac{K}{L})^{\frac{\sigma-1}{\sigma}} = 1^{-\infty} = 0$,thus

$$\lim_{\sigma\to 0} \frac{F(K,L)}{L} = \lim_{\sigma\to 0}(1-\gamma)^{\frac{\sigma}{\sigma-1}} = 1$$

$$\lim_{\sigma\to 0} F(K,L) = L$$

If $L \geq K$

$$\lim_{\sigma\to 0} \frac{F(K,L)}{K} = \lim_{\sigma\to 0}(\gamma + (1-\gamma)(\frac{L}{K})^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}} = 1$$

$$\lim_{\sigma\to 0} F(K,L) = K$$

Thus,

$$\lim_{\sigma \to 0} F(K, L) = min\{K, L\}$$

Capital and labor represent Leontief technology when $\sigma \to 0$.

When $\sigma \to 1$, I need to take the natrual logarithm on both sides

$$\lim_{\sigma \to 1} lnF(K, L) = \lim_{\sigma \to 1} ln[\gamma K^{\frac{\sigma-1}{\sigma}} + (1-\gamma)L^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} = \lim_{\sigma \to 1} \frac{\gamma K^{\frac{\sigma-1}{\sigma}} + (1-\gamma)L^{\frac{\sigma-1}{\sigma}}}{\frac{\sigma-1}{\sigma}}$$

Because both numerator and denominator approach to 0 when I take the limit, L'Hospital rule can be applied. Take the derivative of both terms,

$$\lim_{\sigma \to 1} \frac{\gamma K^{\frac{\sigma-1}{\sigma}} + (1-\gamma)L^{\frac{\sigma-1}{\sigma}}}{\frac{\sigma-1}{\sigma}} = \lim_{\sigma \to 1} \frac{\frac{\gamma K^{\frac{\sigma-1}{\sigma}} lnK + (1-\gamma)L^{\frac{\sigma-1}{\sigma}} lnL}{\gamma K^{\frac{\sigma-1}{\sigma}} + (1-\gamma)L^{\frac{\sigma-1}{\sigma}}}}{\frac{1}{\sigma^2}} = \gamma lnK + (1-\gamma)lnL$$

Thus,

$$\lim_{\sigma \to 1} F(K, L) = K^{\gamma} L^{1-\gamma}$$

This is the form which often appears in the textbook, which also appears in the prototype model (Hosoe, 2004)[16] instead of the original form of constant elasticity of substitution.