SUBSTITUTABILITY OF FINANCIAL ASSETS FOR THE U.S. LIFE INSURANCE COMPANIES : A SIMULTANEOUS EQUATIONS MEAN-VARIANCE UTILITY APPROACH

by

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B.B.A. Acadia University

Thesis

submitted in partial fulfilment of the requirements for

the Degree of Master of Arts in Economics

Acadia University

Fall Convocation 1989

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7 July 1989

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ACKNOWLEDGMENTS

I wish to express my sincere thanks and appreciation to both Dr. A. Hasan and Dr. R. A. Ffrench for their untiring support throughout the course of this thesis development. Dr. Hasan was particularly helpful and patient especially during the early months of the study when the extent of my knowledge was very limited. Both of them read each chapter carefully and made numerous suggestions that have substantially improved both the concept of the work and the clarity of expression. Their efforts are greatly appreciated. A special vote of thanks goes to Mr. J. Qureshi, who took time off from other duties and interests to help me with the programming.

I would like to extend my warmest thanks to Dr. M. Tugwell for his helpful comments on the content of this thesis. As always, I would like to express my gratitude to my family and my friends for their endless encouragement and support.

ABSTRACT

In recent years, the role of life insurance companies has become increasingly important and it now constitutes a major industry in the United States. Policy-holders have entrusted considerable sums of money to life insurance companies with the latter becoming major suppliers of funds to the capital market. As major suppliers of funds, the life insurance companies thus can exert a certain degree of influence on the U.S. economy through a change in their investment behaviour. It is this latter consideration, the investment behaviour of the life insurance company, which constitutes the main focus of this thesis.

This study focuses on the estimation of own and cross elasticities for the financial assets/liabilities of major U.S. life insurers. The methodology for the study is based on a synthesis of portfolio theory and the use of flexible functional forms in demand-system analysis.

The empirical tests reveal that the quadratic utility function generally performed best with the data available. After determining the "optimal" flexible functional form, the estimated mean and variance elasticities for financial assets (liabilities) demand were derived. Finally, a comparison was made of the investment strategies of stock and mutual life insurers. The results would tend to support the belief that mutual life insurers take higher risks in their portfolio selection as compared to the stock life insurers.

CHAPTER 1

INTRODUCTION

Risk is a dimension that researchers, statisticians and analysts have represented as measurable, yet it can be viewed as a component of uncertainty, and uncertainty is one of the fundamental facts of life. For decades, mankind has been exposed to many serious hazards such as fire, disability and premature death, the occurrence of which is impossible to foretell or prevent. However, it is possible to provide some protection against these consequences such as the loss of property or earnings. One of the primary functions of insurance is to serve the purpose of eliminating the risk of loss for the individual. This is not to assert that insurance removes the risk of misfortune, since the mere fact that a person is insured is no guarantee that he will not lose his life. What insurance does do is to provide full or partial compensation to the insured beneficiary upon the occurrence of death or some other specified event. In a sense, insurance provides a specific guarantee against the uncertainty of risks.

Insurance can be effected in numerous policy forms, such as fire insurance, marine insurance, property insurance, liability insurance and life insurance. Among the varied forms of insurance policies, life insurance is considered to be one of the fastest growing financial industry in the United States (see Table 1.1). Given the fact that Americans abhor the thought of leaving their loved ones with inadequate financial resources, a powerful marketing and sales force has bought the life insurance industry to a point where it now holds more than $650 billion in assets, with over $12700 billion of insurance in force, and receives income at the rate of nearly $176 billion in the year of 1983.

According to the OECD's report, the United States has by far the largest amount of life insurance in force in the world. About about $94 billion worth of life insurance was purchased by Americans 1984 alone and almost 80% of the households had at least one type of life insurance coverage. The life insurance premiums which hold the top position among the OECD countries, amount to 43% of the world share of premiums (see Table 1.2).

In accordance with the increasing popularity of the life insurance policy, the number of companies which provide life insurance coverage in the United States has also increased rapidly in the past decades (see Table 1.3). At the end of 1925, there were 158 life insurance companies; however, the number of active companies had increased to 2082 by the end of 1983. The statistics in Table 1.3 also show that about 90% of the operating life insurance companies were owned by stockholders; the remainder were mutual organizations.[1] Mutual companies however, accounted for about 70% of the industry's assets.[2]

The significant growth of the life insurance industry since 1945 can be seen by examining the increase in asset holdings and insurance premiums and annuity income received by the industry (Table 1.4 and 1.5). The pace of expansion has been further accelerated by the government's bias in favour of life insurance. In the late 1950's the U.S. government's change of policy granting tax relief to policy-holders

TABLE 1.1

SELECTED TYPES OF INSURANCE POLICIES IN TERMS OF PREMIUMS PAID
(IN MILLIONS OF DOLLARS)

	1970	1975	1980	1981	1982	1983
Life Insurance	25400	39501	63258	73825	85444	81169
Auto Liability	8958	13315	23319	24395	26226	28080
Workmen's Compensation	3492	6186	14239	14616	13945	14005
Homeowners multiple-peril	2565	4729	9821	10780	11747	12512
Liability other than Auto Liability	2140	3981	7692	7385	7159	7247
Fire	3147	3691	4784	4817	4836	4608
Inland marine	812	1266	2291	2428	2510	2649
Surety & Fidelity	562	789	1248	1351	1454	1649
Ocean marine	465	861	1065	1127	1101	1096
Boiler & Machinery	115	173	293	298	293	356
Burglary & theft	135	120	136	128	115	106
Glass	40	32	32	31	29	27

Source: American Life Insurance 1985 Fact Book, American Council of Life Insurance, Washington, D.C.
Statistical Abstract of the United States, 1987. U.S. Department of Commerce, Washington, D.C.
Figures for life insurance from American Life Insurance 1985 Fact Book and all other figures from Statistical Abstract of the United States. (1987)

TABLE 1.2

DOMESTIC LIFE INSURANCE PREMIUMS PAID WITH WORLD SHARE AND RANK BY OECD COUNTRIES IN 1984

Country	US million dollars	World Share(%)	World Rank
United States	94133	43.48	1
Japan	51756	23.91	2
United Kingdom	15651	7.23	3
Germany	13330	6.16	4
Canada	7192	3.32	5
France	5782	2.67	6
Switzerland	2831	1.31	8
Netherlands	2779	1.28	9
Australia	2579	1.19	11
Sweden	1891	0.87	12
Finland	1222	0.57	14
Italy	1007	0.47	15
Norway	814	0.38	17
Ireland	768	0.35	18
Denmark	760	0.35	19
Belgium	758	0.35	20
Austria	641	0.30	21
Spain	344	0.16	22
New Zealand	309	0.14	23
Greece	80	0.04	40

Source: Consumers and Life Insurance, OECD 1987, Paris, CEDEX.

TABLE 1.3

NUMBER OF U.S. LIFE INSURANCE COMPANIES

Year	Stock	Mutual	Total
1875 and Prior	8	23	31
1876-1925	104	54	158
1926-1950	265	93	358
1951-1960	593	112	705
1961-1970	1015	128	1143
1971-1980	1593	132	1725
1981-1983	1950	132	2082

Source: American Life Insurance 1985 Fact Book. American Council of Life Insurance, Washington, D.C.

TABLE 1.4

DISTRIBUTION OF ASSETS OF U.S. LIFE INSURANCE COMPANIES

(IN MILLIONS OF DOLLARS)

Year	Bonds	Stocks	Mortgages	Real Estate	Other	Total Assets	Growth Rate
1917	1975	83	2021	179	1683	5941	-
1920	1949	75	2442	172	2683	7320	23%
1925	3022	81	4808	266	3361	11538	57%
1930	4929	519	7598	548	5286	18880	63%
1935	5314	583	5357	990	9972	23126	23%
1940	8645	605	6636	2065	13515	30802	33%
1945	10060	999	16102	857	26245	44797	45%
1950	23248	2103	29445	1445	21122	64020	42%
1955	35912	3633	41771	2581	18861	90432	41%
1960	46740	4981	60013	3765	22319	119576	32%
1965	58244	9126	74375	4681	26820	158884	33%
1970	73098	15420	75496	6320	38041	207254	30%
1975	105837	28061	89167	9621	56618	289304	9.8%
1980	179603	47366	131080	15033	106128	473210	11%
1981	193806	47670	137747	18278	128302	525803	10%
1982	212772	55730	141989	20624	157048	588163	12%
1983	232123	64868	150399	22234	184724	654948	11%

Source: American Life Insurance 1985 Fact Book. American Council of Life Insurance, Washington, D.C.

<u>TABLE 1.5</u>

PREMIUM RECEIPTS BY U.S. LIFE INSURANCE COMPANIES
(IN MILLIONS OF DOLLARS)

Year	Life Insurance Premium	Annuity	Total Premium Receipts	Total growth Rate
1911	626	4	630	-
1915	776	6	782	24%
1920	1374	7	1381	76%
1925	2340	38	2378	72%
1930	3416	101	3517	48%
1935	3182	491	3673	4.4%
1940	3501	386	3887	5.8%
1945	4589	570	5159	32.7%
1950	6249	939	7188	39.3%
1955	8903	1288	10191	41.0%
1960	11998	1341	13339	30.8%
1965	16083	2260	18343	37.5%
1970	21679	3721	25400	38.5%
1975	29336	10165	39501	55.5%
1980	40829	22429	63258	60.1%
1981	46246	27579	73825	16.7%
1982	50800	34644	85444	15.0%
1983	50625	30544	81169	-5.0%
1984	51274	42859	94133	15.9%

Source: American Life Insurance 1985 Fact Book. American Council of Life Insurance, Washington, D.C.

on premiums paid precipitated a tremendous boom in the life insurance industry.

Many life insurance policy-holders accumulate considerable equity interest in the process of purchasing life insurance protection, and this equity constitutes a significant financial asset on the policy-holders' balance sheets. In turn, life insurance companies are provided with large sums of funds to invest. The wisest choice of the investment of these funds may constitute a major problem, and investment decisions by life companies exert a considerable influence upon the national economy.

The foregoing discussion demonstrates the growing importance of the life insurance industry in the United States. In considering their role as collectors of savings, we note that life insurance companies account for a significant percentage of savings per family. This definitely establishes them as the most important institution with a fiduciary responsibility for individual savings. Furthermore, the volume of life insurance, in particular, is steadily growing. The assets of the insurance companies have an average growth rate of 10% per annum for the past 10 years and there is no evidence, as yet, of any slackening in the pace of expansion. Thus, it is very likely that the insurance industry will continue to control a large portion of the household savings for many years to come.

Motivated by these conditions, this thesis will attempt to investigate empirically the investment behaviour of some of the large insurance companies in the United States. The investment behaviour of life insurance companies can be analyzed from a number of perspectives. However, of primary interest to us is the manner in which the life insurance companies make their portfolio selections. The study of the life insurance companies' investment behaviour has attracted a great deal attention in recent years. Most of the studies, however, are concerned with the application of quadratic programming techniques and the construction of efficient sets rather than with the utility maximization approach (mean-variance). With the belief that the utility-dependent approach to portfolio analysis could potentially lead to a more powerful result, Aivazian, et. al (1983) developed a two-parameter portfolio model by combining the elements of utility and insurance theories. The significance of this latter model is its ability to simultaneously determine the efficient composition of insurance and investment activities of the life insurance company. For instance, legal, quantitative, and qualitative restrictions on portfolio composition, tax laws, risk, expected costs, and expected returns are all elements that could be dealt with simultaneously within this model.

The comparative static analysis also allows one to estimate the elasticities of substitution between financial assets and liabilities. Specifically, with such an approach, one would be able to estimate the following two types of elasticities:

(i) the impact of a change in expected return of asset/liability A on the insurer's demand for asset/liability B and,

(ii) the impact of a change in the riskiness (i.e. variance) of asset/liability A on the demand for asset/liability B.

In order to detect the insurer's responsiveness toward a change of policy in terms of the insurers' portfolio selection, the estimation of the substitutability among assets within life insurers' portfolios is of potential importance to the relevant authority[3]. The impact of risk-reduction regulations in the equity market on the demand for other securities is one such example. The use of Seemingly Unrelated Regression (SUR) techniques allows one to carry out all the equation estimations simultaneously which provides better results as compared to other single equation studies.

Krinsky (1985) used a similar utility maximization model to examine the Canadian life insurance companies' investment behaviour for the period 1945 to 1977. Since the statistics indicate that the United States life insurance companies play a major role as the suppliers of funds to the financial markets, it is important to undertake the study of life insurance companies using the United States data. In this thesis, we have used the systems (SUR) approach to study the investment behaviour of the American life insurance companies from the period of 1953 to 1983.

The thesis is organized into six chapters. The topic is introduced in the current chapter. As life insurance companies have made available a large number of policies to fit practically any need for life insurance that may arise, it is important to know the principles underlying these basic contracts. Chapter 2 will discuss the issues related to the various types of policies, including term insurance, whole life insurance, endowment insurance and industrial life insurance, annuities, as well as the distinctive features of life insurance contracts. In Chapter 3, a description of the creation of funds for investment by life insurance companies will be provided. The "level" premium concept is introduced to show how a life insurance company resolves its dilemma in allocating funds to meet the increase in claims in the later years. A simple calculation of the premium rate is introduced in order to provide a better understanding of the life industry. Chapters 2 and 3 give an overall view of how insurance companies generate their funds for investment.

Chapter 4 provides a survey of the literature on investment behaviour of the life insurance companies. This survey serves two purposes. First, it provides a background on the theory; and second, it sets the stage for a discussion in the next chapter of the findings in this study in the light of questions raised and observations made in previous studies. Discussion of the model to be used for estimation purposes is also included in this chapter. The central thrust in chapter 5 is the empirical analysis and interpretation of the results. The methodology of the estimation as well as the limitations of such estimation is examined. The final chapter of the thesis presents a summary of the issues with some conclusions about possible reforms.

ENDNOTES

1. A stock life-insurance company is one which is organized by stockholders who subscribe the necessary funds to launch the business, whereas a mutual life-insurance company is a cooperative association of persons established for the purpose of effecting insurance on their own lives.

2. Eight of the 10 largest insurance companies were mutuals. This information was compiled from company data by S.I.C. industries (1984).

3. For example, the tax authority would like to examine the investment behaviour of the life insurance companies before considering any change in government policy.

CHAPTER 2

ALTERNATIVE LIFE INSURANCE CONTRACTS

2.1 INTRODUCTION

The rapid growth of the insurance industry has led to keen competition among the insurance companies. Many of the insurance companies, to distinguish themselves from their competitors, have developed contracts containing special innovative features. Nonetheless, despite the great variety of life insurance policies that exist, the principal forms of life insurance contracts may be broadly classified into: (a) life insurance and (b) annuity insurance. Each of these two broad categories can in turn be divided into different individual contracts containing different features and provisions.

The purpose of this chapter is to discuss the nature and functions of those respective policies. This awareness of the principles underlying the different contracts will enable us to understand how funds are generated by the life insurance companies for investment purposes.

Section 2.2 undertakes a discussion of life insurance contracts with specific examination of individual, group and industrial contracts. Section 2.3 discusses the other broad category of life insurance which is the annuity contract. A brief comparison between the different types of contract is also provided. A short summary is given in Section 2.4 to conclude the chapter.

2.2 Life Insurance

Broadly speaking, life insurance contracts consist of three main bodies, that is, **individual**, **group** and **industrial** life insurance. The individual life policy provides protection to the insured or his family for unforeseeable circumstances, whereas group life insurance serves the purpose of protecting people at work. The industrial life policy is designed to protect the interest of the lower income group who could not afford to carry any other kinds of life insurance policies. Each branch of the life insurance contract will now be discussed in turn.

2.2.1 Individual Life Policies

The three major policies provided by the individual life industry are **term insurance**, **whole life insurance** and **endowment**.

2.2.1.1 Term Insurance

Term insurance may be defined as life insurance under which a fixed sum is payable if death should occur within a given period of time stated in the policy, and nothing is paid in the event of survival. Its sole purpose is to provide temporary protection against a possible loss.

There are many different durations of term insurance whereby the insured may be covered for a period of 1 year, 3 years, 5 years or any other term that is agreed upon. The premium for term insurance is relatively low as the contract only covers a contingency, not a certainty. Except for a long period coverage, such as the age of 65, most policies issued will not become payable because the probability of death is less likely to occur for a short period of time. Hence, the cost of insurance is low.

Term insurance is written in a variety of ways. The insurance protection may be "level"[1], that is the amount of the death benefit may be fixed, or it may increase or decrease over the period. Most insurance companies provide term policies which are renewable at the option of the insured. When a term policy is renewed, there is no need for any medical examination or other evidence of insurability. However, the premium will rise with each renewal at the attained age. For those insured, at the expiration of term, many may find themselves unable to obtain any other form of life insurance protection due to changes in their physical condition, occupational hazards; or other reasons. This renewable feature is of great value to them. In order to prevent any adverse selection at a later age, most the insurance companies are unlikely to extend the renewal feature beyond the age of 65, or 70 years.

Another important feature of the term policy is convertibility. The policy is exchangeable for a permanent plan regardless of the insured's state of health at the time of exchange. Conversion may be made either currently or retroactively, that is either as of the date of the exchange or as of the original date or some intermediate date of the term policy. If the term policy is converted as of the current date, the premium rate of the new contract is at the current rate of his present age. If the conversion is effective as of the original date, the premium rate for the permanent contract would be started at the date of issue of the term policy. An adjustment on account of the differences in past premiums with interest would be required in order to put both the insurer and the insured in approximately the same financial situation as if the permanent policy has been acquired originally.

Normally, a retroactive conversion privilege must be exercised within a limited time before the expiry of the term policy. This is to protect the company against adverse selection arising from the policy-holder's poor health at or near the end of the term period. Some companies, in attempting to reduce adverse selection, offer automatic convertible policies which are automatically converted to a specified plan of permanent insurance at the end of a given number of years. Since the policy-holders in poor health are more likely to continue with a permanent plan than those enjoying good health, the effectiveness of such a policy remains doubtful.

2.2.1.2 Whole Life Insurance

In contrast to term insurance, whole life insurance provides protection for the entire life time of the insured.[2] In other words, a fixed sum is payable upon the death of the insured, and not if the insured dies within a stated period of time. Whole life insurance includes **ordinary life**, **limited-payment life**, **single-premium life** and **joint-life policies**.

Ordinary Life : The ordinary life policy introduces the combination of investment and protection at the lowest annual premium. The policy is issued on a level premium basis and the premium will continue until the death of the insured. In other words, the insured makes excess payments in the early years and these excess payments are accumulated as saving for the policy holders. At the same time, the life insurance companies are provided with funds for investment.

The ordinary life policy is useful if the insured wishes to accumulate a saving fund upon retirement, or provides premature death protection. The flexible provision in the policy also allows the insured to discontinue the payment of premium at any time without forfeiting the cash value which has accumulated under the policy. If, for example, an insured wishes to discontinue premium payments after his retirement, say at age 65, he could exercise his right to convert his original life insurance to a paid-up-at 60 contract prior to his retirement, and the only price is the reduction in the amount of protection.

Limited-Payment Life Insurance : Limited-Payment policies are those in which premiums are limited by contract to a specified number of years. With the payment of last premium, the limited-payment policy is fully paid up. In other words, no premiums are required from the insured despite the fact that the policy is still in force for the rest of his life. The amount of insurance is payable, as in the case of the ordinary-life policy, upon the death of the insured.

The value of a limited-payment insurance is precisely the same as the ordinary life contract, except for the fact that each premium payment is larger than the comparable premium under the ordinary life contract due to the shorter paying period. However, the higher premiums are offset by greater cash and other surrender values.[3]

Limited-Payment insurance may be useful to people who have a short earning career, as in the case of a professional athlete or dancer.

The insured will know in advance that he has purchased an adequate amount of limited-payment insurance to meet his protection needs.

The matter of selection of ordinary life, or the limited payment life policies is largely an individual choice as there is no difference in final benefits between one form and the other.

Single-Premium Life : A single-premium life contract is the extreme form of the limited payment contract. Under this contract the number of premium payment is limited to one. Single-Premium is basically an investment contract for capital accumulation. It obtains a fairly high interest yield with many investment advantages. However, for the purpose of protection, single-premium life insurance is of limited use. It is computed in a way that there will be no refund on any part of the premium in the event of the insured's premature death. Nevertheless, it does serve the needs of people who are interested in investment, or people who are looking for a place to put some "windfall income".

Joint-Life Insurance : A joint-life contract is one written to cover one or more lives, and is payable in the event of the first death amongst the lives insured. Due to the practical difficulties and high expenses, many companies do not cover more than three lives in a contract.

A joint-life policy may be written on any form of life insurance, such as whole policies, limited-payment policies, endowment, etc. Due to the attractiveness of separate term insurance, the life insurance company never issues joint-life on term insurance. Term insurance is preferable to a joint-life policy as the former would offer the advantage of continued protection on the survivors, and it only costs a little more than the joint policy.

A joint-Life policy is particularly appealing to those business partners who may wish to have protection against any financial losses resulting from the death of any one of the partners. A joint-Life policy may also be suitable for a husband and wife where the death of one or the other will create a need for funds.

2.2.1.3 Endowment policies

The endowment policies combine the features of a term policy and a pure endowment policy.[4] Under such a contract, a fixed sum is payable at the end of a specified period if the insured is still living or upon his premature death. Endowment policies approximate an ordinary-life or limited-payment life policy if the maturity date is in the distant future. According to the mortality table on premium payment, an ordinary-life policy is actually considered as an "endowment at age 100".[5]

The endowment policies may be divided into:

(a) a long-term endowment that matures at a specified age such as fifty-five, sixty or seventy;

(b) a short-term endowment that matures in a specified number of year such as ten, fifteen or twenty.

An endowment policy is usually a vehicle for saving and accumulation of funds over a period of time. One of the most popular uses of an endowment policy is to accumulate funds for the education of a child. Another popular usage is to provide funds for retirement. Of all the life insurance policies, the endowment carries the largest investment element with little insurance protection. Because of the heavy savings element, many individuals are facing the dilemma of not having enough funds to continue for death protection upon endowment age. Generally, the endowment policy is only appealing to those with limited premature death protection need and those with a greater need for a specific cash fund at a future date.

2.2.2 Group Life Insurance

The second broad branch of life insurance is group life insurance. It is not only newer, but it is also one of the fastest growing sectors as compared to either individual or industrial life insurance sectors. Group life insurance provides protection for the lives of an entire group of employees under a single contract. The single contract is called the "master policy". Under this plan, a master policy is issued to the employer and the employee receives a certificate detailing information such as the amount of protection, the name of the beneficiary and the privileges of convertibility. Premiums are paid by the employer and the cost may be shared between employer and employee or paid entirely by the employer. To a degree, group life insurance is made available to the participating employees without a medical examination or other evidence of insurability. Because of the large volume of operations (through mass distribution), group life insurance is able to provide low-cost protection. The group life insurance is generally of a continuing nature. The contract will continue beyond the life time of any individual despite the addition of new persons or the coming and leaving of employees from time to time.

The group life insurance policies are further classified into three broad categories:

Yearly Renewable Term Insurance : The term insurance is issued under a group contract to employers, creditors, unions, associations and other eligible entities. The principles underlying group term insurance are the same as those underlying individual life insurance. With respect to each participating employee, the protection expires at the end of each year, but is automatically renewed for another year without evidence of insurability. Similarly, the premium rate will increase each year at the attained age. The policy can be terminated by the employee if he gives notice of thirty-one days prior to the termination of his services with the company.

Group Permanent Life Insurance : This group of contracts provides for some form of accumulation of permanent or cash value units. It may further be divided into the group paid up and the level-premium group contracts.

The group paid up involves a combination of single-premium whole life insurance with decreasing term insurance. It involves both the employees and the employer contribution in premium payment. The purpose of group paid-up insurance is to provide continued protection beyond the employee's retirement age. The employee may retain the permanent insurance upon his retirement or if he wishes, may take a cash-surrender value based on his own contributions in lieu of permanent insurance.

Under the group level-premium plans, the insurance may be in the form of a whole-life, endowment or retirement-income plan other than term policy. Upon the termination of employment, the employee will have cash or paid-up privileges or he may continue the entire amount of the coverage in force by paying the full premium directly to the company.

Special Forms of Group Life Insurance : One of the special forms of life insurance payable upon death as a consequence of accident is often referred as "group accidental death and dismemberment" insurance. This coverage may be both occupational and unoccupational. The other forms of group insurance are employer group life, and association group life, and federal employees group life. These are often referred to as wholesale life insurance plans for groups which are too small to qualify for group coverage.

"Survivors Benefit Group Life" is another form of coverage where an amount is paid to the insured's surviving spouse or children upon the death of the insured.

2.2.3 Industrial Insurance

Up to this point, the discussion of life insurance has been concerned with insurance purchased in face amounts of $1000 or more, with the premium payment at the office of the insurance company. The third broad branch of life insurance is concerned with industrial life insurance. These policies are for amounts less than $1000, with premiums payable at short intervals on a weekly or monthly basis. The collection of these premiums is accomplished at the home of the insured.

This form of life insurance is especially designed for low-income families, mainly belonging to the industrial classes, who could not afford to carry the ordinary life-insurance policies.

The distinctive feature of industrial insurance is the small coverage sums with premiums usually payable weekly. It is generally written on all member of a family unit from birth to an age of 65 or 70. As the amount of insurance involved is small, the policy is written without medical requirement. For many years, industrial insurance has been the only policy available to cover the lives of young children.

Industrial policies offer many of the same types of policies as ordinary policies. The three most basic plans over the years have been : (a) whole life paid-up at 65 or 75; (b) Twenty-year life insurance; (c) Twenty-year endowment policies.

The continuous-premium whole life plan was introduced in the early years of industrial insurance. It was later found that the payment of premiums at advanced ages was too heavy for the industrial policy-holder. As a result, limited-payment policies were introduced. Among the policies, twenty-year endowment insurance policies have been especially popular for children. Many companies have discontinued writing short-term endowments issued on a weekly premium basis. Instead, policies are issued on the monthly premium plan which is less expensive.

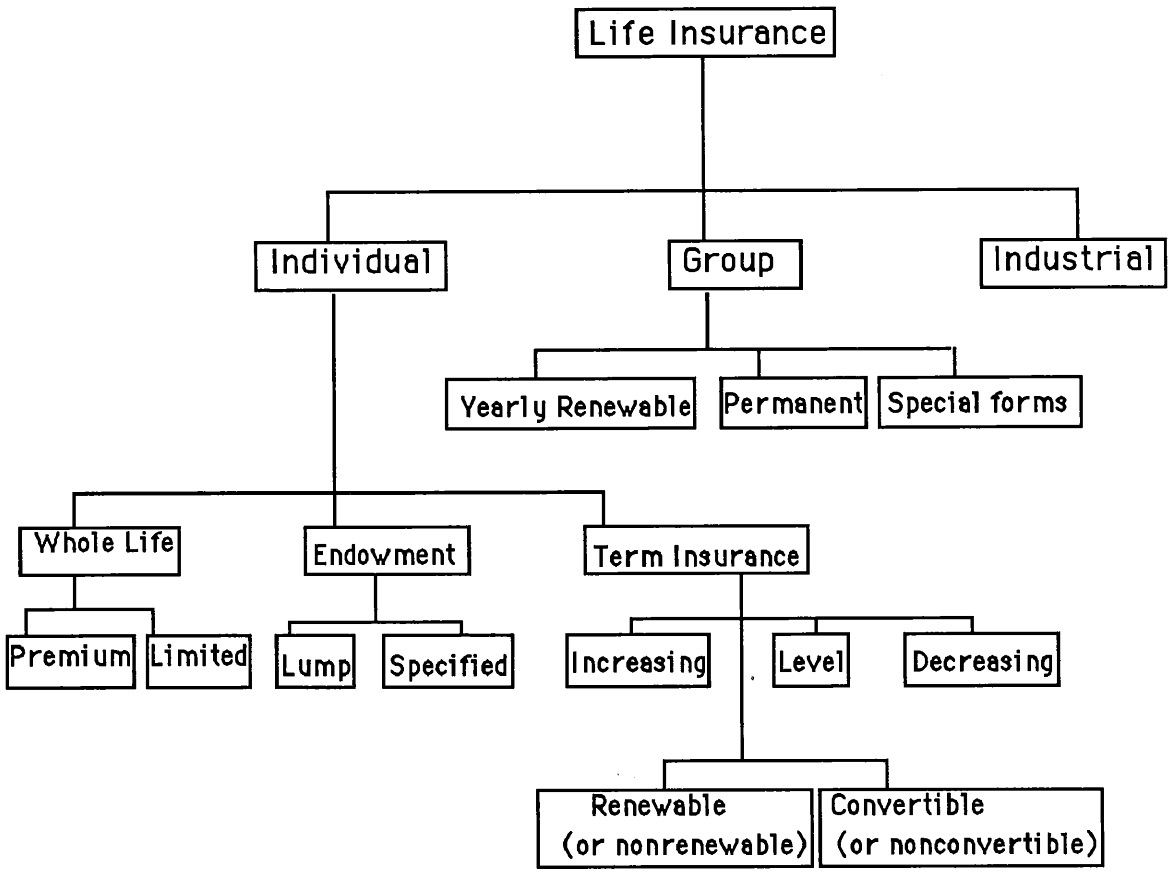
A summary of the different kinds of life insurance policies are presented in Figure 2.1.

2.3 Annuity

An annuity is a contract in which a periodic payment is made to the owner of the contract (the annuitant) commencing at a stated time or age and continuing throughout a fixed period or for the remainder of the owner's life time.

FIGURE 2.1

TYPES OF LIFE INSURANCE CONTRACTS



Source: Gregg, W. D., & Lucas, V. D., "Life and Health Insurance Handbook," Dow Jones, Irwine Inc (1973)

An annuity is often referred to as operating on "the reverse of the life insurance principle". Such a statement is based on the notion that the primary function of life insurance is to accumulate an estate or principal sum whereas the primary function of an annuity is to liquidate a principal sum. In layman's terms, an annuity stops the payment upon the death of the insured and the life contract starts the payment upon the death of the insured. At first glance, the annuity concept seems to be the opposite of the life insurance one; however, upon closes examination of the fundamentals, it is seem that they both provide protection against loss of income. Life insurance can be said to provide protection in the event of premature death and while an annuity can be said to provide protection for longevity.

An annuity may be of various kinds, depending on the type and form of the contract. Basically, it can be classified as follows: (a) method of paying premium; (b) disposition of proceeds; (c) date benefits begin; (d) number of lives covered; (e) units in which pay-out benefits are expressed.

2.3.1 Method of Paying Premium

Annuities may be purchased either by single premium or periodic premiums. An annuitant who pays a lump sum in return for a regular income for life or for a term is provided with a single premium annuity.

The single-premium annuity is widely used in qualified pension and profit-sharing plans. Its main purpose is to ensure that employees have a certain amount of income upon their retirement.

The annual-premium is one whereby premiums are paid in periodic instalments over the year prior to the date on which the annuity income begins. The annual-premium contract offers a greater flexibility as compared to the single-premium. The annuitant is allowed to cease payment anytime and select a paid-up annuity which reduced the amount of protection. The annual-premium is useful for those who treat an annuity as a savings contract.

2.3.2 Disposition Of Proceeds

Under this classification, the annuities may be further divided into a life annuity with the following features: (a) no refund; (b) guaranteed minimum annuity; (c) an annuity certain; (d) a temporary life annuity.

The life annuity with no refund : This contract is frequently referred to as "straight life annuity" which provides an income to the annuitant for life. The annuity is fully liquidated upon the death of the annuitant regardless of how many payments have been received. Because of this no-refund feature, the surviving annuitant is able to enjoy the largest income payment per dollar of purchase price.

Guaranteed Minimum Annuities : Under this type of annuity, a minimum number of annuity payments is guaranteed. If the annuitant should die before the minimum number of guaranteed payments are made, the beneficiary will be entitled to the remaining portion up to the designated amount. The payments will continue throughout the annuitant's life if he lives beyond the guarantee period. Since the payment will be provided to the annuitant whether he is alive or dead, the policy purchased per dollar is more expensive than the straight-life insurance.

The Annuity Certain : The annuity certain will provide the annuitant with a given income for a specified number of years regardless of whether the annuitant lives or dies. This form of annuity is commonly used as a method of paying out life insurance proceeds to a beneficiary for a fixed period of time. If the first beneficiary should die, the second beneficiary is eligible to receive the payment until the policy is liquidated.

Temporary Life Annuities : Temporary life annuities are similar to the annuity certain except that payments stop upon the death of the annuitant. These annuities are rarely seen as they involve a high degree of uncertainty.

2.3.3 Date Benefits Begin

There are two options available for the way in which an annuitant receives his benefits. First is the immediate annuity in which a single benefit is paid at the end of the first income period and throughout the term. If the payment is only made at the end of a given number of years or at optional ages stated in the contract, the contract is known as a "deferred annuity". The deferred annuity may be purchased with either a single premium or by instalment. Similarly, the annuitant has the option of stopping payment anytime under the instalment basis, with a reduction in the amount of protection received.

2.3.4 Number Of Lives Covered

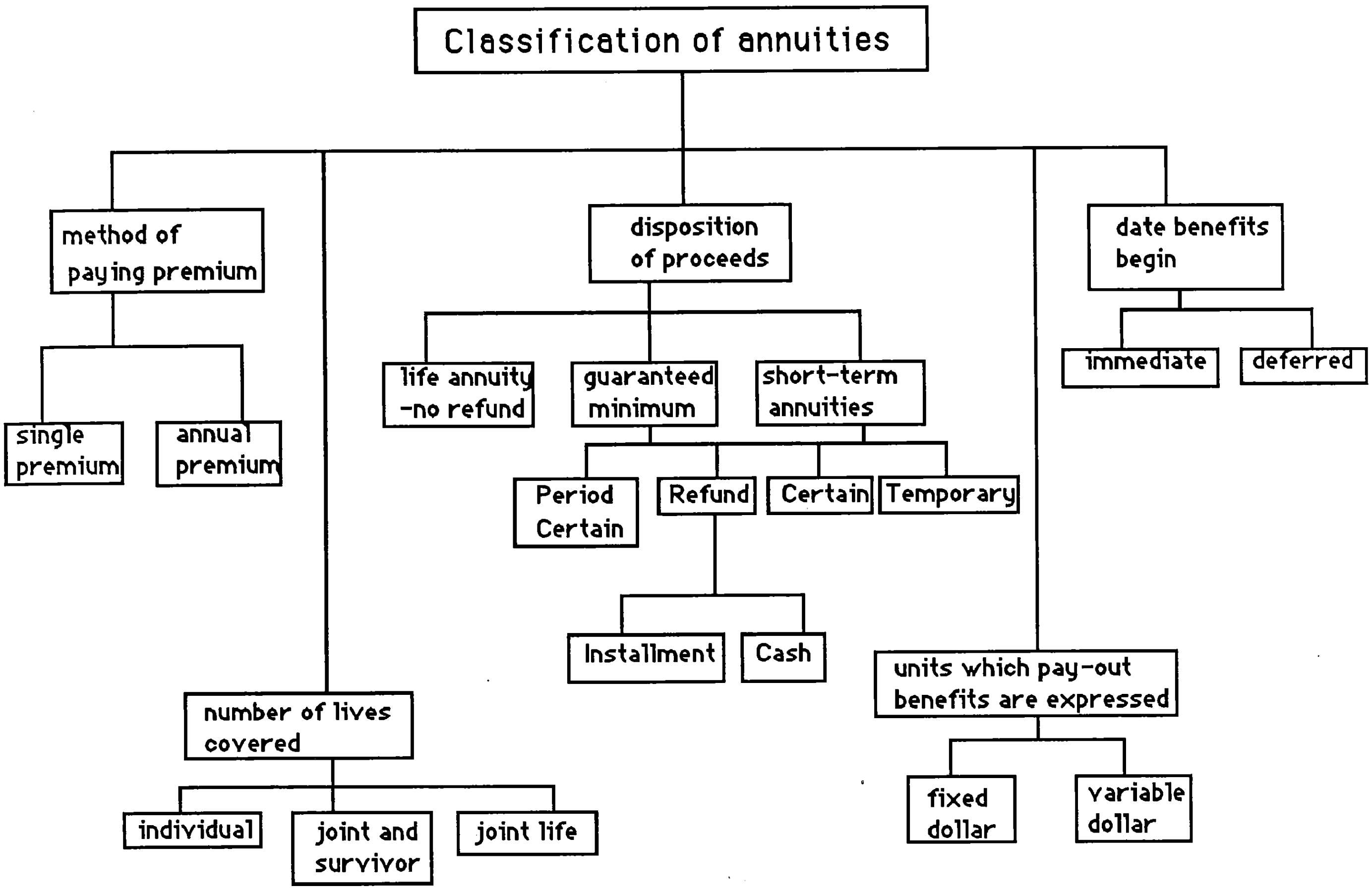
The conventional form of annuity covers only one life. However, it is sometimes issued on more than one life which may be a joint annuity or a joint-and-survivor annuity. A joint annuity is one where payment will cease upon the first death among the lives involved. The joint-and-survivor annuity, on the other hand, will provide payment through the life-time of the last survivor.

2.3.5 Units In Which Pay-Out Benefits Are Expressed

An annuity may be paid as a fixed amount (fixed-dollar) over a specified period, or payment may vary with the changes in the purchasing power of the dollar, which has been termed a variable annuity. If the annuitant is willing to accept the risk of decreasing dollar value brought on by inflation, he is most likely to purchase a fixed-dollar annuity. Otherwise, a variable annuity will provide him with a better opportunity to maintain his purchasing power.

A summary of the different kinds of annuities is presented in Figure 2.2.

FIGURE 2.2
TYPES OF ANNUITIES



Source: Mehr, R. I. & Osler, R. W., "Modern Life Insurance,"
New York (1967)

2.4 Summary

The foregoing discussion gives us an overall view of the various types of policies in both the life insurance and the annuity category. Among the wide range of life insurance products available, we observed that term insurance is the form which contains the largest protective element, whereas the other policies such as whole life, endowment, or annuities contain more saving elements. The implications of these different provisions in term of the generation of investment funds, will be discussed in the following chapter.

Generally, these different life insurance policies are likely to be changed and modified according to changes in government policy.[6] In our discussion, thus far, none of the life insurance policies noted is mutually exclusive. Every policy will contain the various features from each class, such as a single-premium policy may be combined with a refund-fixed dollar annuity. These various combinations of policies will thus be able to meet special needs or circumstances.

In the next chapter how these contracts help to create investment funds for the life insurance companies will be examined.

Endnotes

1 The "level" premium concept will be discussed in more detail in chapter 3.

2. Whole life insurance is also known as Permanent life insurance.

3. Surrendering a policy may be one option open to a holder who decides to make different investment choices. Most policies acquire a surrender value after two years but, because of commission, expenses and the life cover already obtained, this value can be lower than the total amount of premiums paid if surrender takes place within the first ten years.

4. A pure endowment is a life insurance contract where payment is made if the insured survives to the end of the period with nothing being paid in the case of prior death.

5. See Mcgill, D. M., "Life Insurance," Homewood, Illinois (1967), pg. 59.

6. If the government changes their policy by granting tax relief to only a certain type of life insurance policy, it is likely that life insurance companies would change their product to suit the change in policy.

CHAPTER 3

ACCUMULATION OF FUNDS BY LIFE INSURANCE COMPANIES

3.1 Introduction

In chapter 2, the different types of life insurance policies which provide various mixtures of protection and savings were discussed. It is the savings element that allows life insurance companies to acquire investment funds. The issues of how the life insurance companies incorporate the savings element into the policy in order to obtain the investment funds and how the life insurance companies accumulate their investment income to meet the insured's claims in the later years will be discussed in this chapter.

The chapter is organized as follows: Section 3.2 presents a discussion of the sources of funds. Since the investment funds are not sufficient to sustain the life insurance companies' solvency, it is necessary to introduce the concept of the "level" premium which is discussed in Section 3.3. Section 3.4 deals with the process of accumulation and decumulation of funds by taking whole life insurance and endowment as examples. Since the premium income provides the major fund flow for the life insurance companies, the determination of the premium rate is also a crucial factor for the life insurance companies' financial position. Thus, premium rate-making is discussed in Section 3.5, followed by a short summary in Section 3.6 to conclude the chapter.

3.2 Sources of Funds

Among the various fields of general insurance, life insurance generates the largest amount of funds in terms of accumulation and investment of funds. In 1985, life insurance companies invested 295.3 billion dollars in the United States economy, which is about three times the investment fund accounted for by other forms of insurance.[1] This enormous accumulation of funds pooled by insurance companies can be attributed partly to the technical conditions[2] of life insurance business, and also partly to the public demand for various peculiar forms of investment which life insurance companies are able to provide.

Generally, the main source of investment funds for life insurance companies is derived (a) from the margin of profit, (b) from the interest earned on the excess of payment premiums, and (c) from the income on invested assets.

In the insurance industry, the average profit margin range is between two and four percent of the premium income. The variation of the profit margin is very much dependent on market competition. Sometimes, the profit margin may be negative when insurance companies try to obtain unexpired premiums for investment by attracting new business. As long as any loss in the underwriting operation can be covered by interest earned on the unexpired premium, the insurance companies will still be able to operate on a profitable basis.

Among the various sources of funds, insurance premiums constitute the major fund flow. Premiums comprise almost 75%[3] of the total in-flow income with most of the premium payments generated from individual life insurance. Whether the insurance company is able to maintain a solvent position depends heavily on the company's ability to match the advanced premium payments with the future claims and expenses. This, however, is never an easy task as the basic difficulty arises from the fact that an individual's probability of death increases with advancing age. In other words, the cost of insurance rises steadily with age.

The obvious way out of this difficulty is to provide policies such as yearly renewable term insurance where each group of policy-holders of a given age is considered as a separate class for premium purposes. Each group member pays a share of their own death claims and the burden is distributed among the members of the group. Since the death rate increases with age, the premium for yearly renewable term insurance also increases accordingly.

Although the "increasing premium" method of writing insurance is theoretically sound, it has proved unfeasible in practice. One of the reasons is that the increasing cost of the policy forces many policy-holders to drop the insurance even though they may have a genuine need for it. This defeats the purpose of life insurance. Moreover, it has been found that many healthy individuals tend to give up their protection when the rates are high, and those in poor health struggle to keep renewing their policies, regardless of cost. The trend of healthy members dropping out and members in poor health remaining has accelerated the increase in requirements for pay-out due to the death rate. In the end, the premium predicted will not meet the claims and the company has to bear the loss. For these reasons, the need for incorporating a level premium into insurance policies has increased.

3.3 The Level Premium Concept

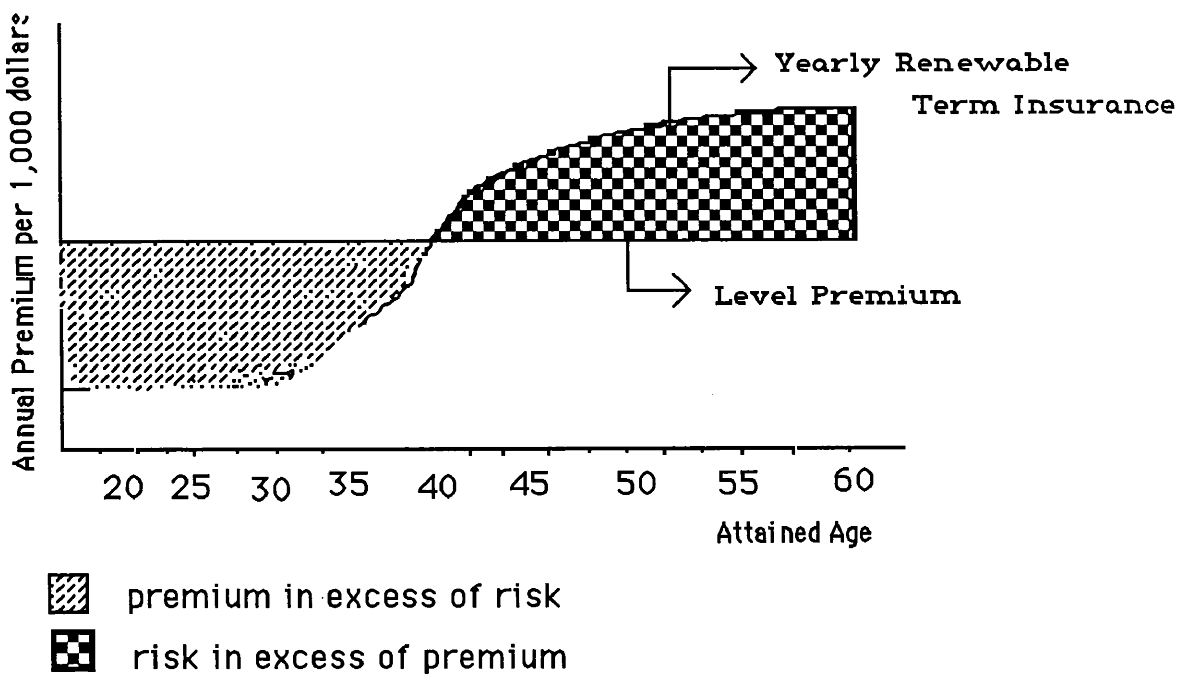
As the name indicates, level premium insurance means that premiums remain constant throughout the premium-paying period instead of rising from year to year as the probability of death increases. Such an arrangement means that the premium income in the earlier year of policies will be in excess of current claims plus expenses, and funds will therefore accumulate gradually through interest earning in order to meet the heavier claims of the later years. The nature of this process is shown in Figure 3.1.

From the figure, the relationship between the level of premium and the yearly rising premium method can be seemed clearly. The significance of the level premium involves the retention of the redundant premiums in the early years of the contract and the restoration of these premiums in the later years.

The "level premium" concept has been incorporated into the various forms of life insurance contracts which provide an effective means of accumulating savings. This form of contract can be found in the majority of individual life insurance policies. Instead of providing protection alone, the actuaries have produced a large variety of policies with the combination of both protection and savings elements. The saving elements vary in degree according to the type of the policy. Figure 3.2 depicts the nature of the relative savings accumulation in the basic type of individual life insurance.

FIGURE 3.1

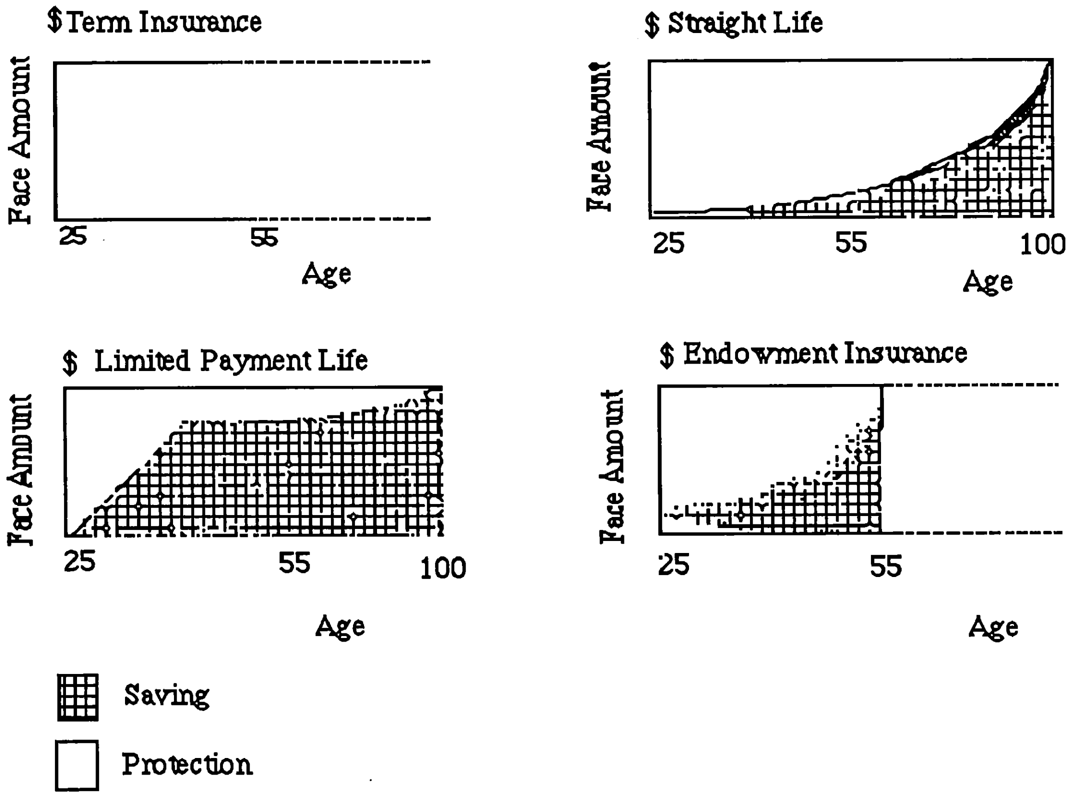
COMPARISON OF YEARLY RENEWABLE TERM PREMIUM WITH LEVEL PREMIUM



Source: Clayton, G., & Osborn, W. T., "Insurance Company Investment," London, George Allen & Unwin Ltd (1965)

FIGURE 3.2

THE NATURE OF THE RELATIVE SAVINGS ACCUMULATION IN THE CASE OF INDIVIDUAL LIFE INSURANCE



Source : Gregg, D. W., & Lucas, V. B., "Life and Health Insurance Handbook," Dow Jones, Irwin Inc (1973)

The above diagrams show that limited payment whole life has a more rapid accumulation of savings as compared to straight life or endowment insurance. In contrast, term life insurance mainly emphasizes protection with no savings element at all. Therefore, any trend toward term insurance policies will decrease the pool of funds administered by the life insurance companies.

Thus, the level premium technique allows the life insurance companies to obtain the excess premium as investment funds, to earn profit for covering the increase in claims in the later years. Without the excess premium, it would be difficult for the life insurance companies to remain at solvency level. We will present the overall process of accumulation and decumulation of funds by taking the whole-life policy and endowment policy as examples.

3.4 The Process of Accumulation & Decumulation of Funds

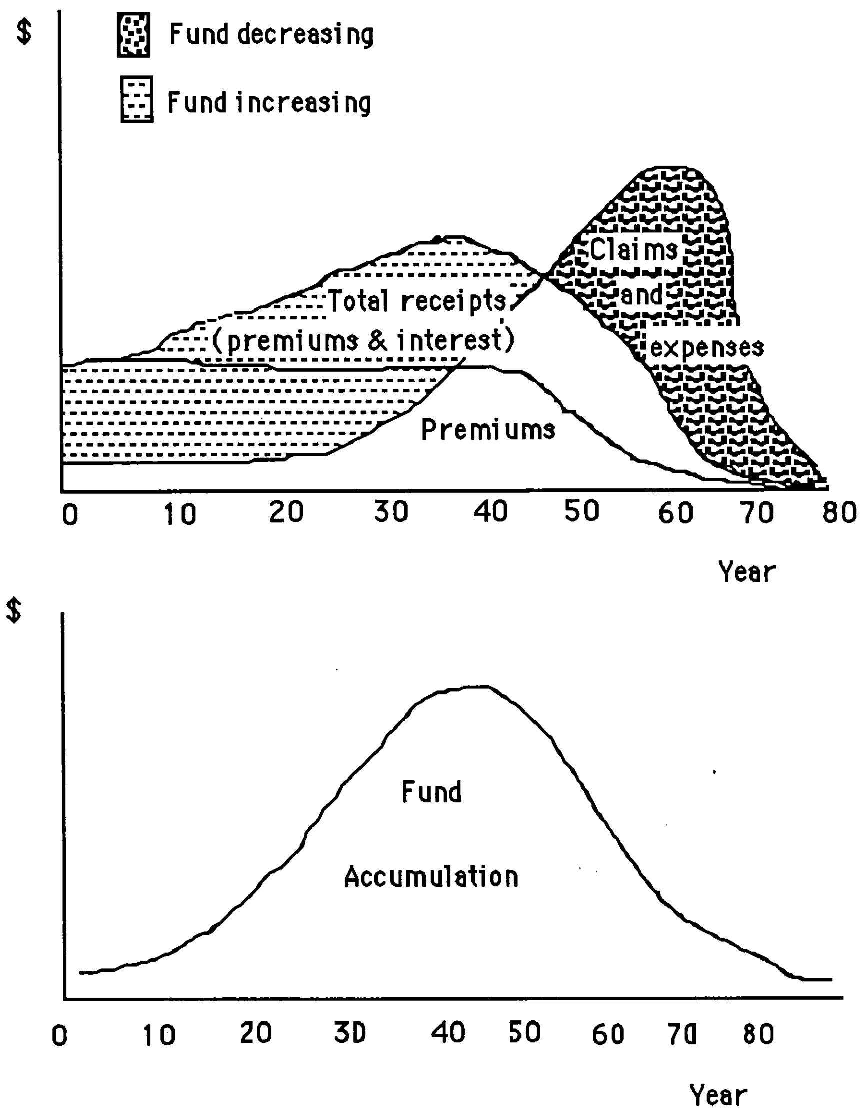
If we consider a group of whole-life policies as an example, the whole process of fund accumulation can be explained simply. The excess premiums accumulate at compound interest. The fund gradually accumulates to a peak and then a decumulation of funds takes place where the premium falls off again. The process is shown at Figure 3.3.

A similar situation arises in the case of the endowment policy as shown in Figure 3.4. As a result of increasing death rates when the policy holders grow older, the total amount of claims paid out grows larger each year and the total premiums collected every year gradually decreases. However, because interest earnings are increasing every year based on the accumulation of funds in past years, the total amount of funds is still rising despite the decrease in premiums.

The technical process of fund accumulation and decumulation is, however, very complicated as it depends upon the actuaries' abilities to fix the rate of premium and its growth at the assumed rate of interest, in order to meet the outflow of claims and expenses. In practice, a high degree of accuracy in determining the amount of claims is possible, as it can be calculated with the assistance of mortality-tables which take into account all the principle factors such

FIGURE 3.3

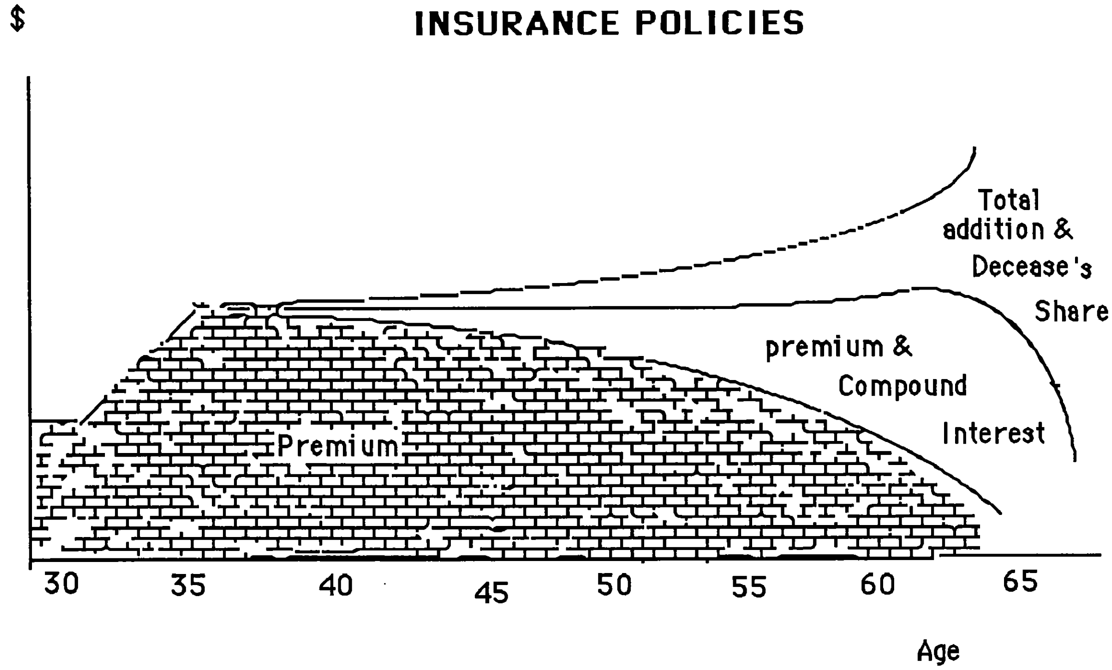
RELATION BETWEEN RECEIPTS AND EXPENSES FOR 1000 WHOLE LIFE POLICIES



Source: Clayton, G., & Osborn, W. T., "Insurance Company Investment," London, George Allen & Unwin Ltd (1965)

FIGURE 3.4

GROUP OF 30-YEAR LIFE ENDOWMENT INSURANCE POLICIES



Source: Huebner, S. S., "Life Insurance," New York, D. Appleton and Company (1960) as age, sex and class of life. The real difficulty arises from the inability to predict the future level of interest rates. To minimize risk taking, most insurance companies assume a lower interest rate in their insurance contract than reasonably expected in order to provide an additional margin of safety. Since the determination of the premium rate is a crucial factor for the life insurance companies to maintain a solvency level, a brief discussion of the mathematical basis for premium computation is presented in Section 3.5.

3.5 Premium Rate-making

Premiums are computed on the basis of information relating to these important aspects of the insurance calculation : (a) the rate of mortality; (b) the rate of interest; (c) the rate of expenses.

To simplify the sample calculation, the expenses relating to overhead, commission, taxes and so on have been ignored. We have also ignored margins for contingencies and profits. Only the assumed rate of mortality and the assumed rate of interest are taken into account. This means that we only calculate the net premium instead of the gross premium. The computation of premium involves both "compound interest" theory and the "present value" concept. An illustration of both the compound interest and present value are given before proceeding with the rate-making example.

Compound Interest : The phrase compound interest means that money is never left idle and as soon as the interest is received, it is immediately invested to earn additional interest. For example, with the assumption that the rate of interest is 3% per annum, \$1.00 will accumulate to \$1.03 at the end of year one. If the total amount is left to accumulate for another year, the interest earned will be $0.0309 (3% of $1.03) and the accumulated amount at the second year will be $1.0609 ($1+$0.03+$0.0309). By the end of year three, $1.09273 has accumulated, and so on. The process of interest earnings on interest is similar to the expanding snowball rolling through the snow. The results of these continuing processes are shown in Table 3.1, column (a).

Present Value : Table 3.1 shows that $1 will accumulate to $1.03 at the end of one year at a 3% assumed interest rate. One may say that $1 is the present value of $1.03 payable at the end of one year. Similarly $1 is the present value of $1.0609 payable at the end of two years and so on. On the other hand, instead of expressing $1 as a present value, we could express $1 as a future value and discount it back to the present value. For example, we noted that the present value of $1 payable at the end of one year as 1.00/1.03 or .97087. If we are considering one dollar payable at the end of two years, the present value is 1.00/1.0609 or .9426 and so on. These present values for different periods of time are shown in Figure 3.5.

TABLE 3.1

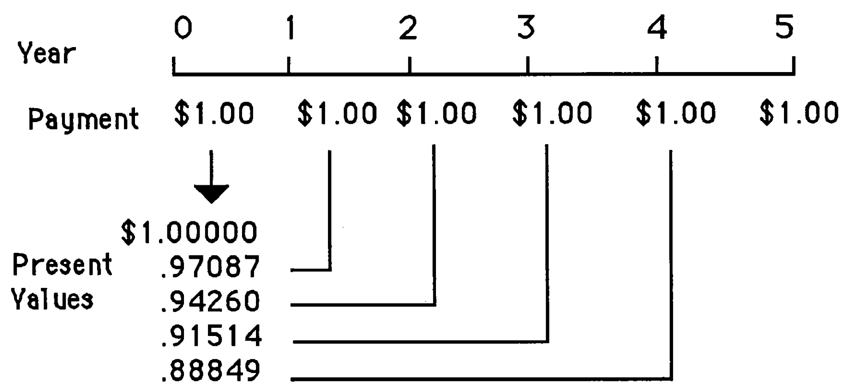
COMPOUND INTEREST TABLE

Number of Years	Accumulated amount of one dollar to an end of year (a)	Present value of one dollar due at an end of year (b)
1	1.03000	0.97087
2	1.06090	0.94260
3	1.09273	0.91514
4	1.12551	0.88849
5	1.15927	0.86261
10	1.34392	0.74409
15	1.55797	0.64186
20	1.80611	0.55368
25	2.09378	0.47761
30	2.42726	0.41199

Source : Pedoe, A., "Life Insurance Annuities and Pensions," University of Toronto Press (1964)

FIGURE 3.5

PRESENT VALUE OF ONE DOLLAR PER ANNUM PAYABLE IN ADVANCE, RATE OF INTEREST 3% PER ANNUM



Computation of the premium rate : The net premium, accumulated at the assumed rate of interest of 3% per annum, with the lives insured subject to an accurately predicted mortality rate will be exactly sufficient to pay the sums insured as they fall due only if the present value of the future premiums is equal to the present value of the future benefits. A five-year term insurance policy, subject to the mortality rate from Table 3.2 is taken as an example.

TABLE 3.2

1958 COMMISSIONS STANDARD ORDINARY MORTALITY TABLE

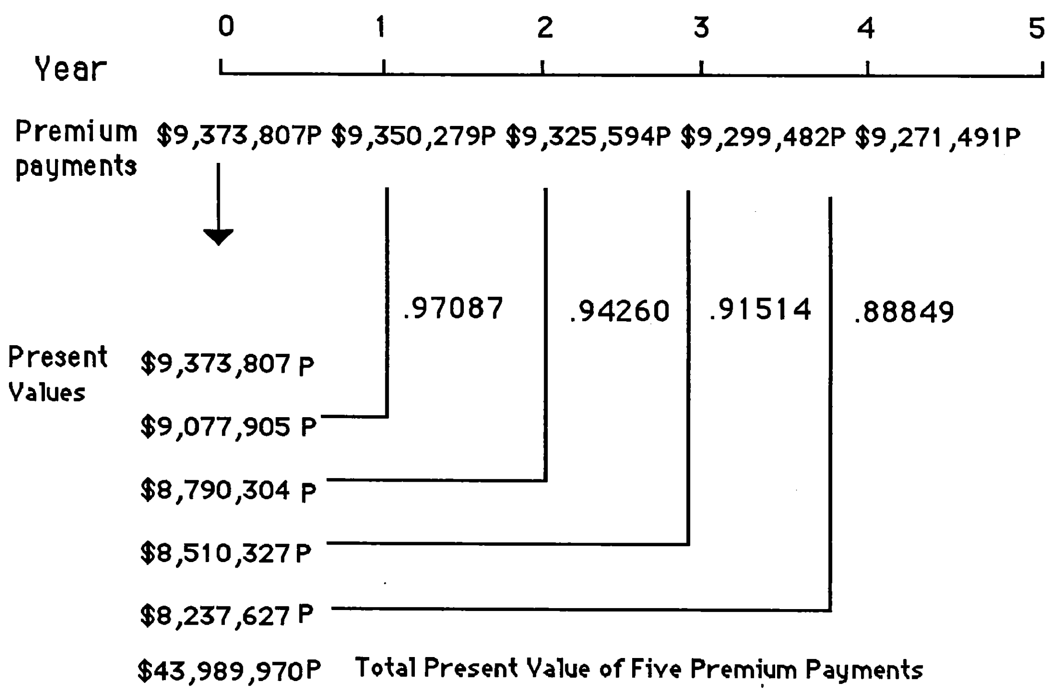
Age	Number Living	Number Dying
35	9,373,807	23,528
36	9,350,279	24,685
37	9,325,594	26,112
38	9,299,482	27,991
39	9,271,491	30,132
40	9,241,359	32,622

Source : Gregg, D. W., & Lucas, V. B., "Life and Health Insurance Handbook," Dow Jones, Irwin Inc. (1973)

Present value of the Annual Premium Payment : Let us assume that 9,373,807 persons age 35, each purchased a life insurance policy for $1000, the net premium for which was P dollars. The premium payable in the first year will be $9,373,807P. In the second year, there are 9,350,279 survivors who will pay the total amount of $9,350,279P and so on. By discounting the second to fifth years premium back to the present (the discounting factors being taken from column b of Table 1), the present value of the five years' payment will be $43,989,970P.

FIGURE 3.6

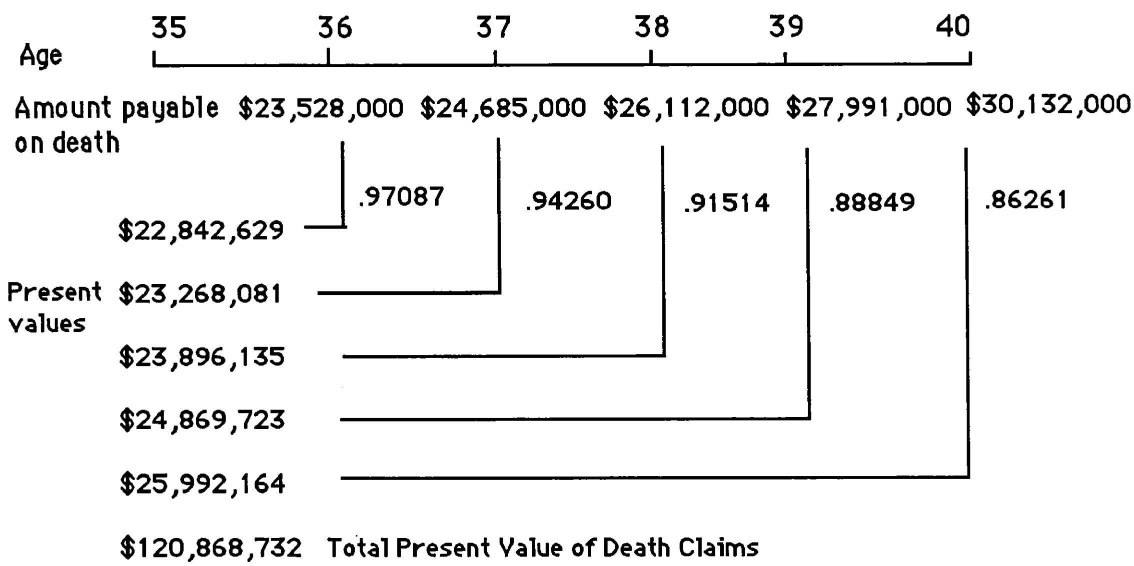
PRESENT VALUE OF ANNUAL PREMIUM OF P DOLLARS PAYABLE IN ADVANCE, FIVE YEAR POLICY, ENTRY AGE 35



Present value of the death claims : Let us consider an amount of $1000 payable on each death during the five year term period. From Table 3.2, we have noticed that there are 23528 deaths claims in year one, therefore, a total amount of $23,528,000 is payable. Similarly, in the second and third year, an amount of $24,685,000 and $26,112,000 are paid respectively and so on. By discounting the future claims back to the present, the total present value of benefits will be $120,868,732.

FIGURE 3.7

PRESENT VALUE OF BENEFITS OF FIVE-YEAR TERM INSURANCE POLICY, ENTRY AGE 35



Net annual premium for the five year term insurance: By equating the present value of the future premiums with the present value of the future death claims, for a five-year term insurance agreement, the rate of premium will be :

present value of net premiums = present value of death claims

$$43989970 \; P = 120868732$$

$$P = 2.747$$

In other words, the net yearly premium for a five year term insurance policy for $1000, with entry at the age of 35 is $2.75.

3.6 Summary

From the above discussion, we noted that the major fund flow is generated from the premium payment. Hence, the determination of the premium rate is crucial to the life insurance company's financial position. There is always some difference between the theory and practice in the computation of the premium rate as the trend of interest rate is highly unpredictable. In practice, a constant adjustment of the net premium for expenses and contingencies is required, in order to maintain a solvent position. By knowing how the investment funds are generated through the various life insurance contracts, we can now proceed to examine the manner in which the life insurance industry utilizes the funds for investment.

Endnotes

1. Board of Governors of the Federal Reserve System Annual Statistical Digest.(1986)

2. One of the technical conditions is the selling life insurance the form of a long term contract.

3. American Life Insurance 1985 Fact Book.

CHAPTER 4

INVESTMENT BEHAVIOUR OF LIFE INSURANCE COMPANIES : SOME SELECTED MODELS

4.1 Introduction

We have seen from the previous chapter that the cash available for investment by life insurance companies comes from the premium income, interest income and retained surplus. These diversified sources of funds, together with the equity capital, can be invested in financial markets. The life insurance companies, like other financial institutions, have to face the intermediate risks of matching the cost of liabilities with the returns on the assets investment. If the amount of liabilities is greater than the amount of assets, the life insurance company will be insolvent. On the other hand, if the amount of invested assets is greater than the amount of liabilities, the life insurance company will be able to generate a surplus.

The major threats to a life insurance company's survival arise from two types of investment risks:

(i) capital-value risk; the market value of a security may depreciate over time;

(ii) income-risk; interest income may change during the course of future premium receipts, and/or at the time securities mature and the funds need to be reinvested.

A great deal of attention has been given to these problems. This has led to a number of studies that explore the investment behaviour of the life insurance institution.

The purpose of this chapter is to examine how the life insurers behave in order to overcome their potential solvency problems. A summary of the various approaches is presented in Section 4.2, and a brief discussion of the shortcomings of the various models is undertaken to justify the latter use of the Mean-Variance Utility maximization model. Section 4.3 deals with the criterion of the utility function based on mean-variance analysis, followed by the formulation of the utility maximization model in Section 4.4. The generalized Box Cox flexible form is used to operationalize the utility function introduced in Section 4.5., followed by a concluding summary in Section 4.6.

4.2 Summary of Selected Models

The research done on life insurance companies' investment behaviour in recent years can be divided into two different groups. One group explores the technical implication of "matching" which in the literature is called **hedging** or **segment markets hypothesis**. The other group centers on the concept of "expected yield" **portfolio analysis**. The following section will discuss the two approaches mentioned above in greater detail.

4.2.1 Hedging or Segmented Markets Hypothesis

Hedging is essentially a method of reducing the uncertainty of the availability of needed funds in the future. Haynes and Kirton (1953) argued that this method can be made possible through the concept of "matching". In their paper, they suggested that financial institutions are able to reduce the intermediary risks if the maturity composition of the assets portfolio matches the maturity composition of liabilities. In other words, the life insurance companies could match the long term liabilities with assets invested in longer maturities and likewise with the short term liabilities.

Although the concept of matching is theoretically sound, there are many practical difficulties in carrying out such a policy. In their model, Haynes and Kirton assumed that expected mortality and experienced rate always coincide and that all contracts run their full course without the option of surrender or conversion. However, these assumptions are over-simplified and it is less likely to be practical in the real world. The complication of liquidity options and the multiplicity of settlement options incorporated in the life insurance policies make exact matching a very difficult investment strategy.

Further, in their article, Terrell and Frazer (1972) observe that the investor with hedging motives alone runs the risk of involuntarily liquidating some assets. This is due to the unpredictable change in interest rates whereby those securities with shorter maturity can cause income instability and those of longer maturity are subject to loss of principal when sold prior to maturity.

Instead of hedging alone, Redington (1952), Bagely and Perks (1953) suggested the notion of the liquidity-hedging motive. That is, in addition to hedging, a portion of investment funds will be allocated to relatively short-term securities with relatively greater price stability. This will ensure a stable stream of income in case of any mishap in hedging.

Terrell and Frazer (1972) pointed out that the liquidity-hedging approach has induced the maturity distribution of securities to be weighed more heavily on shorter-dated instruments than would occur with hedging alone. Their studies of the maturity distribution of marketable public debt, held by various institutional investors, support the presence of a liquidity-hedging motive. This notion is illustrated in Figures 4.1 (a) and (b).

There are a number of explanations for the figures presented. In the liquidity-hedging model, the flows of funds are determined by planned expenditures and anticipated needs for discharging liabilities, when viewing commercial banks in contrast to life insurance companies as an example. The former institutions are relatively short-term investors because of the uncertain anticipated outlays. The insurance companies, on the other hand, favour long term investments because they can anticipate outlays far into the future. With the specific hypothesis that standard deviation and mean are directly related, we therefore, observe high movements of mean and standard deviation for the insurance companies as compared to the commercial banks. Thus, such data are consistent with the liquidity-hedging notion.

Support for the liquidity motive also comes from a study by Winklevoss and Zelten (1973), who examined the life insurance companies in the United States for the period 1925-1969. Their data suggested that the surplus level of the five largest mutual life insurers has substantially exceeded the historical need for such funds. These results are also corroborated by a more recent work done by Franklin and Woodhead (1980). Their studies revealed that in addition to hedging, the life insurance company could prevent itself from insolvency by relying on:

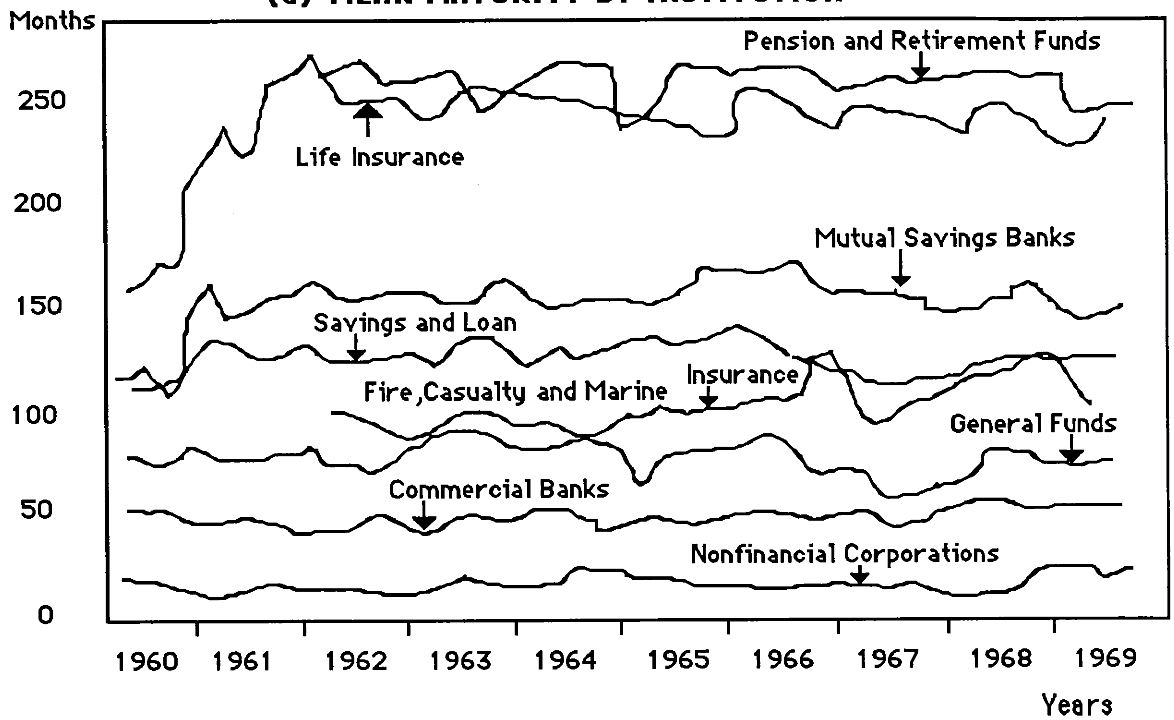
(i) Shareholders capital and undistributed profits.

(ii) Participating contracts.[1]

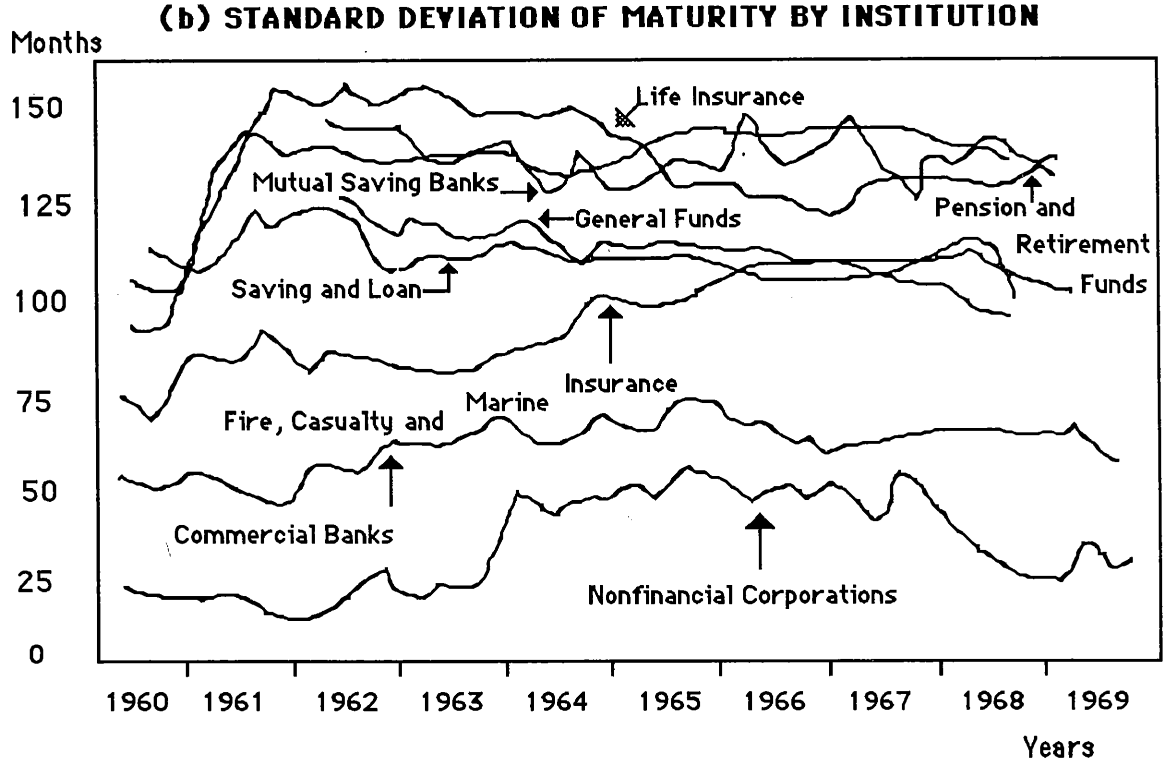
FIGURE 4.1

MARKETABLE INTEREST-BEARING DEBT: MEASURES OF MATURITY PROFILES

(a) MEAN MATURITY BY INSTITUTION



(b) STANDARD DEVIATION OF MATURITY BY INSTITUTION



Source: Terrell, W.T. & Frazer, W.J, "Interest Rates Portfolio Behavior and Marketable Government Securities". Journal of Finance (1972).

In summary, the above discussion suggests that the desire to avoid insolvency may often cause the life insurance companies to engage in long term investment in order to ensure that the realized return will exceed the contractually guaranteed return. This has also been verified by the empirical analysis, which is also consistent with an observation made by an industrial spokesmen. The following statement is a quote from a life insurance company financial vice president :[2]

> "Life insurance investment as contrasted to most other institutional investment is strikingly characterized by its ability to take the long look . . . to invest for the long term with minimum economic consideration necessary for liquidity and marketability." (pg. 20)

The hedging motive together with a large amount of liquidity reserves enables the life insurance companies to safeguard themselves against insolvency. The hedging hypothesis provides an overall framework in analyzing the investment behaviour of the life insurance companies. Generally, this work has been based on conceptual reasoning with limited empirical testing. It is therefore prudent to examine other types of models which are more quantitative. Such models are based on "expected yields" (mean) and the " variance of the expected yields" (variance) to analyze the investment behaviour of the life insurers.

4.2.2 The Mean-Variance Portfolio Analysis

The early literature on portfolio selection [e.g., Pegler (1948), Clarke (1954)] assumed that the investor is able to maximize the expected value of his utility function by the maximization of expected yield (return) alone. Markowitz (1952) rejected this notion of the single investment rule model because it failed to admit a diversified portfolio selection. He suggested that the expected yield should include an allowance for portfolio risk which is measured by the variance of returns. Markowitz believed that investors are essentially risk averters, who will only shoulder more risk(variance) if they are compensated by a gain in expected yield, and in general, they will choose a diversified portfolio. Analytically, this characterization is accomplished by describing each asset exclusively in terms of expected return (E), variance of expected return (V), and the correlation (or covariance) of expected returns between the i_{th} and j_{th} securities (σij).[3]

The expect return E from the portfolio as a whole is indicated below :

$$E = \sum_{i=1}^{N} X_i u_i$$

and the variance is

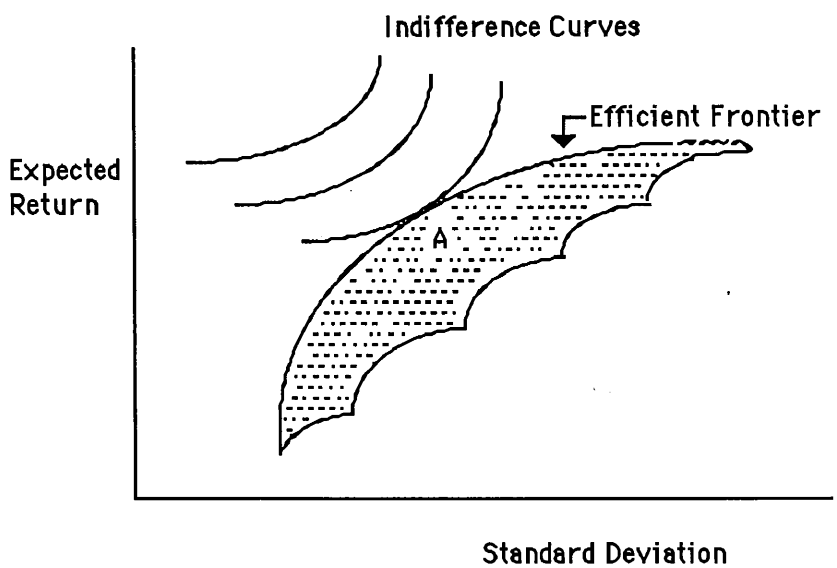
$$V = \sum_{i=1}^{N} \sum_{j=1}^{N} \sigma_{ij} X_i X_j$$

Where Xi is the percentage of the investor's assets which are allocated to the i_{th} security. $\Sigma X_i = 1$, $X_i >= 0$ for i=1,2,3.......
u_i is the expected return on the portfolio,
σ_{ij} is the covariance of X_{ij}.

For fixed probability beliefs, the investor has a choice of various combinations of E and V depending on his choice of portfolios, X_1 .... X_N. However, the E-V rule states that the investor would only want to select one of those portfolios which is on the efficient frontiers as indicated in Figure 4.2; that is, portfolios which satisfy the requirement that no combination of assets can produce a higher expected return without incurring greater variability of return. Markowitz's model has left the specific portfolio selection to the individual investor's preference function. If the investor's preferences are formulated in terms of a utility function, he is assumed to be able to maximize his expected utility through the efficient portfolio which is depicted at point **A** in Figure 4.2.

FIGURE 4.2

MARKOWITZ'S EFFICIENT SETS



4.2.2.1 The Applicability of Markowitz' Model in Relation to the Life Insurance Companies.

In Markowitz's model, attention was concentrated only on the asset side of investment by assuming that the absolute level of investment funds available is fixed, with no effort being directed to the inclusion of liabilities. This is not desirable for the life insurance company where its underwriting activities and the disportion of investment funds are not independent.

Pyle (1971) developed a three-security model in which he analyzed the portfolio problem of intermediaries and the circumstances under which a firm would be willing to sell a given deposit (liability) in order to invest in the financial asset. His empirical results revealed some interesting relationships among a riskless security, loans and deposits. In particular, he found :

1. the smaller the risk premium on deposits and the larger the risk premium on loans,
2. the greater the positive dependence between loan and deposit yields, and
3. the larger the standard deviation of deposit yields and the smaller the standard deviation of loan yields.

Pyle (1971) therefore concludes that,[4]

> "By and large, the literature on the theory of financial intermedation has concentrated on either the asset side or the liability side of the balance sheet. By explicitly considering the dependence between the securities bought and sold by financial intermediaries, it has been shown that asset(liability) portfolios cannot, in general, be chosen independently of the parameters of liability(asset) yields." (pg. 746)

Using data on nineteen insurance lines and two types of assets for the period 1956-1971, Kahane and Nye (1975) estimated the correlations among underwriting profits in various insurance lines, among investment profits, and also between insurance and investment activities. These correlations are presented in the form of a correlation matrix in Table 4.1. Thus, for instance, the correlation coefficient between an asset (bonds, item 20) and a liability (item 6) is negative instead of zero.

TABLE 4.1

CORRELATION MATRIX FOR AGGREGATE STOCK COMPANIES 1956-71

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	1																				
2	.67	1																			
3	-.29	.73	1																		
4	-.33	-.49	.19	1																	
5.	- .3	-.17	-.21	.63	1																
6	-.15	-.34	.60	-.48	-.54	1															
7	.40	.68	-.24	-.17	-.31	-.03	1														
8	.54	.45	-.67	-.08	-.52	-.19	.55	1													
9	-.19	.28	.00	-.01	-.26	.05	.66	.31	1												
10	-.04	-.02	.31	-.46	-.45	.65	-.05	.01	.04	1											
11	-.27	.02	-.27	.50	.52	-.55	-.17	-.21	.07	.27	1										
12	.15	-.04	.03	-.34	-.48	.19	.20	.47	-.12	.39	-.41	1									
13	.52	.47	-.51	-.44	-.23	-.13	.02	.41	-.37	.20	-.17	.61	1								
14	.51	.08	-.08	-.16	-.11	-.21	-.24	.23	-.67	.26	-.29	.23	.51	1							
15	-.52	-.33	.12	.32	.46	-.33	-.38	-.33	-.12	-.01	.27	-.16	-.27	-.17	1						
16	.36	.62	-.68	.06	.15	-.73	.23	.51	-.05	-.24	.24	.18	.56	.36	.14	1					
17	.40	.38	-.16	.03	-.11	-.45	.26	.47	.29	-.23	.18	.16	.18	.19	-.04	.31	1				
18	.39	.10	.07	.08	-.28	.06	.74	-.10	-.04	-.06	.60	.47	.49	.08	.51	.09	.1	1			
19	.09	.67	-.54	-.11	.12	-.17	.75	.02	.61	-.17	.22	-.50	-.08	-.45	-.26	.20	.09	.41	1		
20	.39	.61	-.45	-.50	-.40	-.11	.46	.47	.21	.10	-.27	.19	.38	.03	-.32	.32	.18	-.04	.29	1	
21	.07	.14	.05	-.27	-.49	.09	.24	.17	.21	.03	.02	-.26	-.01	.15	-.04	.10	.11	.01	.11	.27	1

Definitions :

1- Ocean Marine
2- Inland Marine
3- Group Accidents & Health
4- Accident & Health
5- Workmen's Compensation
6- Liability
7- Automobile Liability
8- Collision
9- Automobile Fire/Theft
10- Fidelity
11- Sure
12- Glass
13- Burglary & Theft
14- Boiler
15- Credit
16- Fire
17- Allied
18- Homeowner
19- Commercial Multiperil
20- Bonds
21- Stocks

Source: Kahane, Y., & Nye, D., "A Portfolio Approach to the Property-Liability Insurance Industry," Journal of Risk and Insurance (1975).

Thus both the Pyle and the Kahane and Nye studies suggest that the structure of the assets and liabilities in the life insurance industry cannot be considered independently. The fact that a higher level of surplus appears to be correlated to riskier investment, and that higher costs of reserve liabilities are associated with less risky investment, suggests that the underwriting activities and investment decisions are simultaneously determined. Stowe (1978) also noted that:[5]

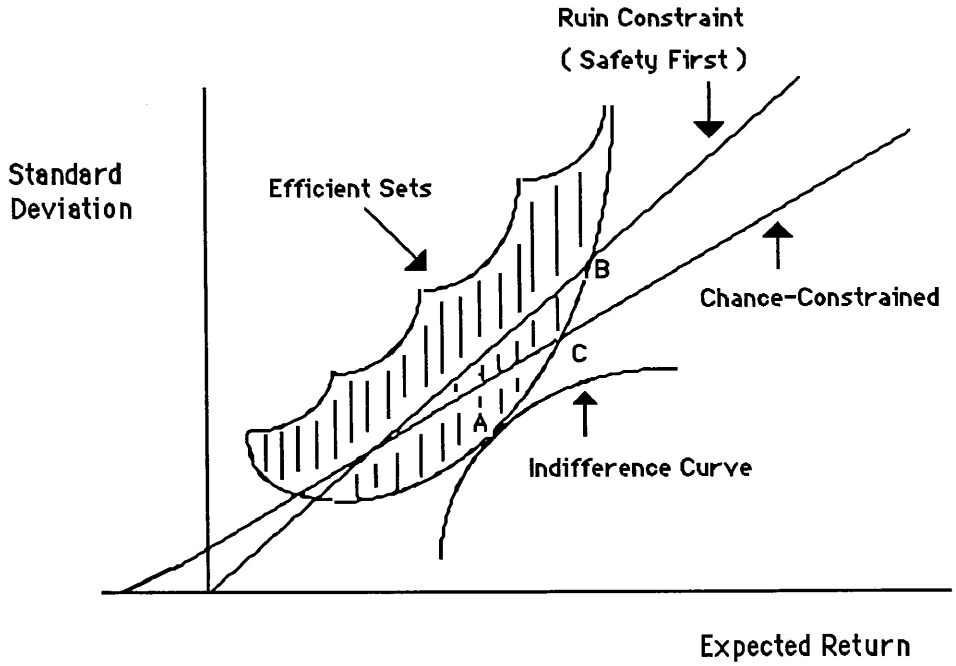
> "This model (Markowitz) was not used because no explicit relationship exists between the amount and cost of the major life insurance company's liabilities and its portfolio choices. Consequently, this model does not yield explicit testable hypotheses." (pg. 435)

4.2.2.2 The Extensions of the Markowitz's Model

Several studies have extended Markowitz's model to incorporate both assets and liabilities. See for instance Lambert (1966), Krouse (1970), Haugen and Kroncke (1970,71). However, while much of this work has contributed towards developing a bounder theoretical framework, only limited empirical testing has been performed. Moreover, these studies focus more on the determination of the set of mean-variance efficient portfolios than on the question of the determination of the optimal portfolio on the efficient sets. Parallel to these developments, several studies explored further the question of locating an operating point on the efficient sets. This operating point is determined by imposing decision rules, which vary according to the approach taken. The three major approaches are : (a) the Utility theory (b) the Ruin (Safety First) constraint (c) the Chance-Constrained model. These various decision rules are depicted in Figure 4.3 :

FIGURE 4.3

CHOICE OF AN OPERATING POINT THROUGH RUIN, CHANCE AND UTILITY-BASED DECISION RULES



Markowitz originally suggested that the expected utility of wealth formulation of the portfolio selection problem could be approximated by considering preference orderings over the mean and variance of the portfolio return. However, it is not at all clear that this form of the mean-variance utility function is an adequate measure of all the relevant dimensions of the portfolio decision problem.[6] Because of the arbitrary nature of the utility function, some economists like Pratt and Arrow (1964) called for the rejection of mean-variance analysis as a criterion for portfolio selection, while other attempted to develop a more objective criteria instead of using expected utility as a yardstick.

One such objective criteria was the safety-first criterion which was originated by Roy (1952). Roy asserts that :[7]

" a man who seeks advice about his actions will not be grateful for the suggestion that he maximizes expected utility." (pg. 433)

He believed that investors are more concerned about the safety of the investment than investment yields. Thus, investors are assumed to have in mind some disaster level of returns and they will react to avoid such a possible disaster.

The major component of the safety-first principle is to define a critical value (outcome) **d**, which measures the gross income from the portfolio held. This critical value may vary among investors. Any income, **e**, which is less than **d**, is considered a disaster. Using the Bienayme-Tchebycheff inequality, Roy obtains :

$$P([e-m] >= m - d) =< \frac{\sigma^2}{(m-d)^2}$$

and a fortiori

$$P(m-e >= m-d) = P(e =< d) =< \frac{\sigma^2}{(m-d)^2} \qquad (4.a)$$

where m is the expected value of the gross return and s is the standard error of e. Since the left hand side of equation (4.a) is simply the upper bound of the probability of disaster, the investor will choose the portfolio investment that minimizes the probability of this event; that is, the portfolio that minimizes $\frac{\sigma^2}{(m-d)^2}$

Pyle and Turnovsky (1970) examined the safety-first criterion in relation to the expected utility maximization and concluded that:[8]

"... as long as there is no risk-less, for any portfolio chosen by an expected utility maximizing investor with concave (μ, σ) indifference curves, we can always find a safety-first investor who will choose the same portfolio. ... If a riskless asset is available then except in one special case the safety-first criterion does not lead to the traditional liquidity preference." (pg.75)

Stowe (1978) provides an alternative explanation of the portfolio choices of the life insurer through the chance-constrained model. A general chance-constrained portfolio model can specified as follows:

$$\text{Max} \quad r = R'X \tag{1}$$

$$\text{s.t.} \quad \Pr[(1+R)'X > (1+k)L] >= \alpha \tag{2}$$

$$X_i >= 0 \text{ for all } i \tag{3}$$

$$X_i =< C_i \text{ for some } i \tag{4}$$

$$\Sigma X_i = 1 \tag{5}$$

where

X = vector of proportions of total assets invested in n securities
R = vector of expected rates of return
r = expected rate of return on total assets
L = total legal reserve liabilities/total assets
C_i = legal maximum constraints
α = probability of solvency
k = rate of return paid on liabilities
i = securities

In this model, the life insurer is assumed to maximize the rate of return on its assets subject to :

(1) a probabilistic solvency constraint;

(2) non-negative constraints;

(3) legal constraints;

(4) and a balance sheet constraint.

The main contribution of this model is its ability to maximize the rate of return on its assets as the total assets plus the expected earning which must at least exceed the total liabilities plus cost. Despite this contribution to the literature, Stowe stated that:[9]

> "... The disadvantage of a chance-constrained model is that it is not a utility maximization model; it is a return maximization model with the degree of risk aversion impound in the solvency probability of the chance-constraint." (pg.435)

In short, the efforts to seek departure from the utility framework by establishing the safety-first and chance-constrained model have failed to justify the basic economic question as to whether the investor is maximizing their expected utility through those decision rules. This state of affairs has provided a compelling reason for the use of the utility framework of mean-variance analysis.

4.3 The Criterion of Using the Mean-Variance Utility Function

As noted above, Markowitz suggested that the investor's preference order could be reflected solely in terms of mean and variance. This notion has been severely criticized by a number of authors such as Borch (1969), Feldstein (1969), Samuelson (1967), and Hanoch and Levy (1979).

Borch (1969) pointed out that any system of upward sloping mean- standard deviation (E-S) indifference curves is incapable of being unconditionally consistent with the logic axiom of choice under uncertainty. For instance, if one tries to combine the model (Markowitz) with the theory of Von Neumann and Morgenstern, there exists a polynomial of degree n utility function. On the other hand, if u(x) is a utility of money function, economic common sense requires that $u'(x)>0$ and $u''(x)<0$; that is, the marginal utility of money is decreasing which implies that the utility function cannot be a polynomial function. Hence this approach to the economics of uncertainty must violate either (a) the consistency of Von Neumann and Morgenstern, or (b) the usual assumption about the utility of money.

Feldstein (1969), by using a log utility function and a lognormal distribution for investment income has shown that E-S indifference curves for a risk-averter need not be convex downwards, though upward sloping. They would change from convex to concave, once the standard deviation of the outcome exceeds the mean multiplied by $1/\sqrt{2}$, which indirectly suggests that risk aversion might decrease as risk itself is increased beyond a certain point.

Samuelson (1967) claimed that in general it is not possible to define a preference ordering of portfolios of mixed investment in term of E and S alone except in the case where they are all normally distributed. Hanoch and Levy (1979) indicated that Markowitz's assumption that risk-averters will diversify only with a larger mean and a smaller variance, is unsound in terms of expected utility. This could be illustrated by an example :

Example 4.1

x	Pr(x)	y	Pr(y)
1	0.80	10	0.99
100	0.20	1000	0.01

$E(X) = 20.8 > E(Y) = 19.9$

$Var(X) = 1468 < Var(Y) = 9703$

It is noted that the Markowitz criterion is satisfied for x. But suppose the utility function is $U(z) = \log_{10} Z$, which is a well-behaved function for all positive values, displaying risk-aversion everywhere. With this utility function, however, $E[u(x)] = 0.4$, $E[u(y)] = 1.02$, and Y is preferred to X.

Tobin and Markowitz (1952, 1969) defend their stand, arguing that if the expected utility maxim is adhered to, the mean-variance analysis is relevant if : (a) the investor's utility function is quadratic, or (b) the distributions of the portfolios are all members of a two-parameter family and the returns are normally distributed. The basis of their claims are discussed in some detail in the following sections.

4.3.1 The Mean-Variance Quadratic Utility Function

Markowitz (1979) claims that if all decision makers show aversion to risk with concave utility functions- that is, functions that incorporate the assumption of diminishing marginal utility of money- the only mathematical form of a utility function which depends on the mean and the variance is the quadratic. This can be illustrated by the following quadratic utility function :

$$U(R) = a + bR + cR^2$$

where a can take any value, while $b>0$ and $c<0$.

The expected utility of R is :

$$E[U(R)] = a + bE(R) + cE(R^2)$$

Since $\sigma^2_R = E(R^2) - [E(R)]^2$

$$E[U(R)] = a + bER + c[E(R)]^2 + c\sigma^2_R$$

Thus, the expected utility in the quadratic case can be expressed as a function of the mean portfolio return. Moreover, the expected utility varies directly with E(R) and inversely with risk.

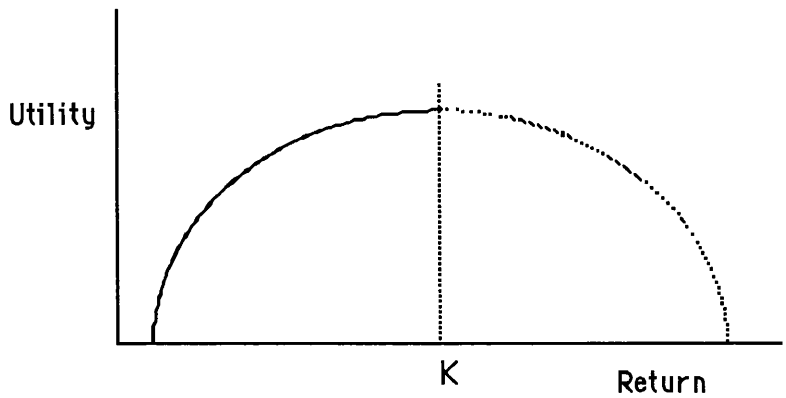
$$\delta E[U(R)] / \delta \sigma^2_R = c<0$$

and $$\delta E[U(R)] / \delta E(R) = b + 2cE(R) >0$$

Thus, investors with diminishing quadratic utility functions will maximize their expected utility by selecting portfolios with the minimum risk for any given rate of return (that is, efficient portfolios).

This quadratic utility function is easy to use, but it is not without its flaws. First of all, it has been argued that a quadratic utility specification is only relevant for a bounded range (the rising portion) for which the marginal utility of additional return is positive. However, after the return has gone beyond K (shown in Figure 4.4), the investor receives a negative marginal utility which contradicts the basic assumption that an investor is rational. Besides, the quadratic utility function shows an increasing degree of risk aversion (measured by $-\mu''[x]/\mu'[x]$),[10] whereas empirical observation as well as theoretical considerations would lead one to assume decreasing (absolute) risk aversion. Therefore, the use of a quadratic utility function is subject to limitations which reduce its usefulness.

FIGURE 4.4
QUADRATIC UTILITY OF RETURNS FUNCTION



4.3.2 The Normal Distribution and Risk Aversion

Instead of restricting the utility function to the quadratic, Markowitz, Tobin and Samuelson [1969, 70] do not rule out the possibility that mean-variance analysis can be justified for a wide class of utility functions by assuming that if the investors are risk-averse, the efficient set will be at its optimum if the rate of return is normally distributed. Rigorous proofs of this result have been carried out by Hanoch and Levy (1969), Samuelson (1970), Fama (1971), and others. A study by Levy and Sarnat (1984), for a sample of 100 mutual funds over the 1959-1980 interval seemed to confirm the hypothesis that the rate of return is distributed normally. Thus, one could presume that a significant proportion of investment choices can be explained by the mean-variance model. If the returns of the individual securities are independent of one another, or at least are not perfectly correlated, the return on relatively large portfolios should approximate a normal distribution. This follows directly from the Central Limit Theorem which states that:[11]

"Let f(x) denote the density function of a random variable with an expected value equal to μ and a variance equal to σ^2 ($\sigma^2 < a$). If X_n denotes the mean of a sample of size n drawn from this distribution then the random variable $(X_n - \mu)/(\sigma/n)$ will approximate a normal distribution with an expected value of zero, and a variance of unity, on the condition that n is sufficiently large." (pg. 37)

In short, the optimal efficiency criteria for the utility function under a two-moment (mean-variance) analysis can be satisfied:

(i) if the investor utility function is quadratic, or

(ii) if the investment outcomes are normally distributed.

4.4 The Utility Maximization Model

The utility-dependent model was first developed by Aivazian, Callen, Krinsky, and Kwan (1983) to study the investment behaviour of the personal sector in the United Kingdom. Krinsky (1985) adopted a similar model for the life insurance sector, but extended it to deal with more complicated problems faced by life insurance companies in Canada. Legal restrictions on portfolio composition, the tax laws, risk, expected costs, and expected returns are all elements that could be dealt simultaneously within the model.

4.4.1 Formation of the Utility Function

Following Krinsky (1985), we assume that there are n possible types of investment and m-n types of life insurance contracts.

Let W_0 = initial wealth

$$\tilde{r}_{it} = \begin{cases} \text{Rate of return on the } i^{th} \text{ investment,} \\ i = 1\ldots n, \text{ in period } t. \\ \text{Rate of return on the } i^{th} \text{ type of insurance} \\ \text{contract, } i = n+1 \ldots m \text{ in period } t. \end{cases}$$

$$l_{it} = \begin{cases} \text{Investment in the } i^{th} \text{ asset in period } t, \\ i = 1\ldots n. \\ \text{Actuarial reserves for the } i^{th} \text{ type of} \\ \text{insurance contract in period } t, \; i = n+1\ldots m. \end{cases}$$

[The random variables are denoted by tildes (-)]

K_{ot} = Policy holder's surplus plus shareholder's equity in period t.

$$K_{ot} = S_{ot} + E_{ot} \tag{4.1}$$

where S_{ot} - Policy-holders' surplus in period t

E_{ot} - Shareholders' equity at the beginning of period t

The profit of the company in period t, $\tilde{\Pi}_t$, is a linear combination of the random variables given by

$$\tilde{\Pi}_t = \sum_{i=1}^{m} l_{it}\tilde{r}_{it} \tag{4.2}$$

In order to obtain the rate of return on equity in period t, $\tilde{\Pi}_{et}$ can be derived by dividing both sides of equation (4.2) by E_{ot}.

$$\frac{\tilde{\Pi}_t}{E_{ot}} = \tilde{\Pi}_{et} = \Sigma \left\{ \frac{l_{it}}{E_{ot}} \right\} \tilde{r}_{it} \tag{4.3}$$

Equation (4.3) can then be rewritten as follows :

$$\tilde{\Pi}_{et} = \sum_{i=1}^{m} W_{it}\, \tilde{r}_{it} \tag{4.4}$$

where

$$W_{it} = \begin{cases} \text{The } i^{th} \text{ asset to equity ratio in period } t, \ i=1,\ldots\ldots,n. \\ \text{Actuarial reserves to equity ratio for insurance} \\ \text{line } i \text{ in period } t, \ i=n+1,\ldots\ldots,m. \end{cases}$$

A balance sheet constraint is introduced into the model in order to ensure solvency. For each and every period the l_i's ($i = 1, \ldots\ldots, m$) must be determined so as to equate total assets with the sum of liabilities. Thus the following relationship must hold:

$$\sum_{i=n+1}^{m} l_{it} + K_{ot} = \sum_{i=1}^{n} l_{it} \tag{4.5}$$

Equation (4.5) represents the balance sheet constraint; that is

total liabilities = sum of actuarial reserves + equity + surplus

= total assets.

Instead of the premium, the actuarial reserves are used for each type of contract. This is to avoid totalling the premium amounts collected in different years. Substituting K_{ot} from equation (4.1) into (4.5) and then dividing equation (4.5) by E_{ot}, we get,

$$\sum_{i=1}^{n} W_{it} = \sum_{i=n+1}^{m} W_{it} + 1 + \frac{S_{ot}}{E_{ot}} \tag{4.6}$$

Further, we define,

$$\frac{S_{ot}}{E_{ot}} = W_{m+1,\,t} \quad \text{(surplus to equity ratio)}$$

Substituting the above definitional equation into equation (4.6) enables us to write the solvency constraint as :[12]

$$\sum_{i=1}^{n} Wi - \sum_{i=n+1}^{m+1} Wi = 1 \qquad (4.7)$$

or

$$\sum_{i=1}^{m=1} Xi = 1 \quad \text{where} \quad Xi = Wi \quad i=1, \ldots, n,$$
$$Xi = -Wi \quad i=n+1, \ldots, m+1$$

The insurance company's investment preferences are assumed to be captured by a utility function defined by the expected (end of the period) net worth E (return) and its standard deviation V (risk):

$$\mathbf{U(E,V)} \qquad (4.8)$$

where $E = W_0 [1 + E(\bar{\Pi}_e)]$

and $V = [W_0^2 \sum_{i=1}^{m+1} \sum_{j=1}^{m+1} XiXjGij]^{1/2}$

where Gij is the covariance of the end of period expected return on asset i and j. The utility function is assumed to be continuous and twice differentiable with $U_E > 0$ and $U_V < 0$. The subscript E denotes the partial derivative of U with respect to E and similarly, the V subscript denotes the partial derivative of U with respect to V. In other words, the insurance company is assumed to be risk averse with indifference curves in the E-V space which are upward sloping and convex from below.

The insurance company is assumed to choose a portion to invest in each financial asset and a portion of underwriting in each type of insurance contract so as to maximize the utility function (4.8) subject to the constraint in (4.7) and the non-negativity conditions

$$Xi >= 0, \; i=1, \ldots m+1 \qquad (4.9)$$

The maximization problem of the life insurer can be stated as:

$$\text{Maximize } U(E, V) \qquad (4.10)$$

$$\text{subject to } \sum_{i=1}^{m+1} X_i = 1 \qquad \text{For } X_i >= 0$$

where

$$E = W_0 [1 + \sum_{i=1}^{m+1} X_i \bar{r}_i]$$

$$V = [W_0^2 \sum_{i=1}^{m+1} \sum_{j=1}^{m+1} X_i X_j G_{ij}]^{1/2}$$

Applying the Lagrangian multiplier method to the above maximization problem, one can obtain the following first-order conditions. A detail derivation of the First Order Condition is given in Appendix 4A.

$$\delta L/\delta X_i = W_0 r_i U_E + W_0^2 U_V V^{-1} \Sigma_j X_j G_{ij} = \lambda \qquad (4.11)$$

$$\delta L/\delta \lambda = 1 - \Sigma_i X_i = 0 \quad i,j = 1........m+1 \qquad (4.12)$$

where λ is the Lagrange multiplier and V^{-1} is $1/V$.

The second order-conditions for a maximum require that the principal minors of the determinant obtained by totally differentiating (4.11) and (4.12) with respect to the Xi, alternate in sign. A detailed derivation is given in Appendix 4.B.

4.4.2 A Comparative Static Analysis of Portfolio Adjustment

The above model enables the derivation of several comparative static results. Of particular interest are the own expected return and variance elasticities of assets/liability demand. In addition, inter-asset(liability) substitutability measures can be obtained by deriving :

(i) the impact of a change in the expected return of asset/liability A on the insurer's demand for asset/liability B, and

(ii) the impact of a change in the riskiness (variance) of asset/liability A on the demand for asset/liability B.

The impact of a change in the j_{th} asset's expected return on the quantity demanded of asset i, holding the Gij elements constant, can be determined by differentiating the system (4.11) and (4.12) with respect to r_r.. This can be shown to yield :

$$\delta X_k/\delta r_r = -W_0 U_E D_{rk}/D - X_r [W_0^2 U_{EE} \sum_{i=1}^{m+1} r_i D_{ik}/D + V^{-1} W_0^3 U_{EV} \Sigma_i \Sigma_j X_j G_{ij} D_{ik}/D] \qquad (4.13)$$

where D is the determinant of the second order own and cross derivation of (4.11) and (4.12) and D_{rk} is the rk cofactor of D (A detail of the derivation is given in Appendix 4.C). The demand elasticity of asset i with respect to the expected return on asset j (i, j=1, ...m+1), is then

$$\eta(X_i, r_j) = \frac{\delta X_i}{\delta r_j} \frac{r_j}{X_i} \qquad (4.14)$$

In order to find out the effect of a change in G_{rf} on X_k , the same methodology is applied by differentiating the first-order conditions in (4.11) with respect to G_{rf} and solving for $\delta X_k/\delta G_{rf}$. :

$$\delta X_k / \delta G_{rf} = -V^{-1} U_V W_0^2 (X_r D_{rk}/D + X_f D_{fk}/D) - V^{-1} X_r X_f W_0^3 [U_{EV} \Sigma_i r_i D_{ik}/D + W_0 V^{-1} U_{VV} \Sigma_i \Sigma_j X_j G_{ij} D_{ik}/D] \qquad (4.15)$$

and the demand elasticities of asset i with respect to the change in the variance covariance matrix will be

$$\eta(X_i, G_{ij}) = \frac{\delta X_i}{\delta G_{ij}} \quad \frac{G_{ij}}{X_i} \qquad (4.16)$$

4.5 The Estimating Equations

In order to proceed with empirical work we adopt a generalized Box-Cox (flexible functional-form) utility function.[13] Theory offers little guidance as to the appropriate functional form. Thus a general Box-Cox utility function has been proposed which includes the generalized Leontief, generalized square root, quadratic, and translog utility functions as special or limiting cases. The choice between them is then made on empirical grounds.

The insurance company is assumed to maximize an "institutional utility function", defined by mean and standard deviation in the following form:

$$\mathbf{U(\delta) = \alpha_0 + \alpha_1 E(\lambda) + \alpha_2 V(\lambda) + 1/2\, \alpha_3\, [E(\lambda)]^2 + 1/2\, \alpha_4 [V(\lambda)]^2 + \alpha_5 E(\lambda) V(\lambda)} \qquad (4.17)$$

where $E(\lambda)$, $V(\lambda)$ and $U(\delta)$ are the Box-Cox transformation functions defined as:

$U(\delta) = (U^{2\delta} - 1) / 2\delta = (U^{2\delta} - 1) / 2\delta$

$E(\lambda) = (E^{\lambda} - 1) / \lambda$

$V(\lambda) = (V^{\lambda} - 1) / \lambda$

Four alternative cases of the general transformation will be considered. In each case, the parameters λ and δ take on different values and thus the institutional utility function specified in (4.17) will assume different flexible functional forms.

Case I. Translog Utility Function

$$\delta,\lambda \text{ ----> } 0: \quad U(\delta) = \ln(U), \quad E(\lambda) = \ln(E), \quad V(\lambda) = \ln(V)$$

Since $\lim_{l \to 0} \frac{(X^{\lambda}-1)}{\lambda} = \ln X$ by l' Hospital's rule.

Substituting the above functions into equation (4.17), we can obtain the translog utility function as follows :

$$\ln U = \alpha_0 + \alpha_1 \ln E + \alpha_2 \ln V + 1/2\, \alpha_3 (\ln E)^2 + 1/2\, \alpha_4 (\ln V)^2 + \alpha_5 (\ln E)(\ln V) \quad (4.18)$$

Case II Generalized Leonlief Utility Function

$$\delta, \lambda \text{ -->} 1/2 \quad U(\delta) = U - 1, \quad E(\lambda) = 2(E^{1/2} - 1), \quad V(\lambda) = 2(V^{1/2} - 1)$$

A similar procedure is applied to obtain the generalized Leontief utility function.

$$U = 2\alpha_3 E + 2\alpha_4 V + 4\alpha_5 E^{1/2} V^{1/2} + (2\alpha_1 - 4\alpha_3 - 4\alpha_5) E^{1/2} + (2\alpha_2 - 4\alpha_4 - 4\alpha_5) V^{1/2} - 2\alpha_1 - 2\alpha_2 + 2\alpha_3 + 2\alpha_4 + 4\alpha_5 + 1 \quad (4.19)$$

Case III Square Root Quadratic Utility Function

$$\delta, \lambda = 1 \quad U(\delta) = (U^2 - 1)/2, \quad E(\lambda) = E - 1, \quad V(\lambda) = V - 1$$

$$U = [\alpha_3 E^2 + \alpha_4 V^2 + 2\alpha_5 E.V + 2(\alpha_1 - \alpha_3 + \alpha_5) E + 2(\alpha_2 - \alpha_4 - \alpha_5) V + 2(-\alpha_1 - \alpha_2 + \alpha_5) + (\alpha_3 + \alpha_4 + 1)]^{1/2} \quad (4.20)$$

This is known as the square rooted quadratic utility function.

Case IV Quadratic Utility Function

$$\delta = 1/2 \quad \lambda = 1; \quad U(\delta) = U - 1$$
$$E(\lambda) = E - 1$$
$$V(\lambda) = V - 1$$

$$U = 1/2 [\alpha_3 E^2 + \alpha_4 V^2 + 2\alpha_5 EV + 2(\alpha_1 - \alpha_3 - \alpha_5)E + 2(\alpha_2 - \alpha_4 - \alpha_5)V + 2\alpha_5 + \alpha_3 + \alpha_4 - 2\alpha_1 - 2\alpha_2 + 1] \quad (4.21)$$

Before solving the utility maximization problem in order to obtain demand (share) equations, it is convenient to derive the expressions for U_E, U_V, U_{EE}, U_{VV}, U_{EV}. These can be derived from equation (4.17).

$$U_E = \delta U/\delta E = [\alpha_1 + \alpha_3 E(\lambda) + \alpha_5 V(\lambda)] E^{\lambda-1} \quad (4.22)$$

$$U_V = \delta U/\delta V = [\alpha_2 + \alpha_4 V(\lambda) + \alpha_5 E(\lambda)] V^{\lambda-1} \quad (4.23)$$

$$U_{EE} = \delta U^2/\delta V^2$$
$$= (\lambda-1) [\alpha_1 E^{\lambda-2} + \alpha_3 E(\lambda) E^{\lambda-2} + \alpha_5 V(\lambda) E^{\lambda-2}] + \alpha_3 E^{2\lambda-2} \quad (4.24)$$

$$U_{VV} = \delta^2 U/\delta v \delta V$$
$$= (\lambda - 1) [\alpha_2 V^{\lambda-2} + \alpha_4 V(\lambda) V^{\lambda-2} + \alpha_5 V^{\lambda-2} E(\lambda)] + \alpha_4 V^{2\lambda-2} \quad (4.25)$$

$$U_{EV} = \delta^2 U/\delta E \delta V = \alpha_5 V^{\lambda-1} E^{\lambda-1} \quad (4.26)$$

We now proceed to derive the demand equation for the share estimation. Using equation (4.11), for any pair of assets i, r (i /= r), we can write :

$$W_0 U_E r_i + U_V V^{-1} W_0^2 \Sigma_{j=1} X_j G_{ij} - \lambda = 0 \quad (4.11a)$$

and

$$W_0 U_E r_r + U_V V^{-1} W_0^2 \Sigma_{j=1} X_j G_{rj} - \lambda = 0 \quad (4.11b)$$

Equate equation (4.11a) with (4.11b)

$$W_0 U_E r_i + U_V V^{-1} W_0^2 \Sigma_j X_j G_{ij} = W_0 U_E r_r + U_V V^{-1} W_0^2 \Sigma X_j G_{rj}$$

$$W_0 U_E (r_i - r_r) + W_0^2 U_V V^{-1} [\Sigma X_j (G_{ij} - G_{rj})] = 0 \qquad (4.27)$$

By rearranging (4.30), we are able to obtain the demand equation :

$$W_0^2 U_V V^{-1} [\Sigma X_j \{G_{ij} - G_{rj}\}] = - U_E W_0 (r_i - r_r)$$

$$\Sigma_j X_j \{ G_{ij} - G_{rj} \} = - \frac{W_0 U_E (r_i - r_r)}{W_0^2 U_V V^{-1}}$$

$$\Sigma_j X_j \{ G_{ij} - G_{rj} \} = - \frac{U_E (r_i - r_r)}{W_0 U_V V^{-1}}$$

or

$$X = \frac{U_E}{W_0 U_V V{-1}} Z^{-1} r^*$$

$$X = K . Z^{-1} r^* \qquad (4.28)$$

where

$$K = \frac{U_E}{W_0 U_V V^{-1}} = \frac{[\alpha_1 + \alpha_5 E(\lambda) + \alpha_5 V(\lambda)] E^{\lambda-1}}{W_0 [\alpha_2 + \alpha_4 V(\lambda) + \alpha_5 E(\lambda)] V^{\lambda-2}}$$

$$Z_{(m+1 \times m+1)} = \begin{bmatrix} G^*21, & G^*22, & G^*23, \ldots\ldots\ldots\ldots & G^*2m+1 \\ \\ G^*m+1,1, & G^*m+1,2, & G^*m+1,3, \ldots\ldots & G^*m+1,m+1 \\ 1, & 1, & 1, & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} X1 \\ . \\ . \\ . \\ . \\ Xm+1 \end{bmatrix}_{(m+1 \times 1)} \qquad r^* = \begin{bmatrix} r1 - r2 \\ . \\ . \\ . \\ . \\ r1 - rm+1 \\ 1/k \end{bmatrix}_{(m+1 \times 1)}$$

and $G^*_{rj} = G_{1j} - G_{rj}$ (r=2, ... m+1)

(j = 1, ... m+1)

In order to estimate the parameters of the system (4.28), we need to include a serially uncorrelated multivariate normal disturbance (v). Thus, the demand system to be estimated is as follow :

$$X \quad = \quad K.Z^{-1}\, r^* + v \tag{4.28a}$$

where $v(t) = \{ v_1(t), v_2(t) \ldots\ldots\ldots\ldots\ldots\ldots v_{m+1}(t) \}$ t= 1,......,T

The share equations corresponding to the translog, the square rooted quadratic and the general Leontief utility functions can be obtained from equation (4.28) by selecting the appropriate λ. Since δ does not appear in the demand equation, the share equations and the share rooted quadratic utility functions are identical since in both cases $\lambda = 1$.

Once the share elements (α_is) in the demand equations are determined, we can then proceed to estimate the effect of change in expected return (r_r) and variance/covariance (G_{ij}) on demand for each asset (liability), and to check for second order conditions. The equations to be estimated are noted in follows :

By substituting equations (4.22), (4.23), (4.24), (4.25), (4.26) into equation (4.13), we can determine the maximum condition of the Second Order Condition :

$$\delta^2 L/\delta X_i\,\delta X_j = W_0^2 r_i r_j\,[\,\alpha_1(\lambda-1)\,E^{\lambda-2} + \alpha_3(\lambda-1)\,E(\lambda)\,E^{\lambda-2} + \alpha_3 E^{2\lambda-2} + \alpha_5(\lambda-1)\,E^{\lambda-2}V(\lambda)\,]$$
$$+ W_0^3\{\alpha_5 E^{\lambda-1}V^{\lambda-2}\,(\,r_i\Sigma_i X_i G_{ji} + r_j\,\Sigma_j X_j G_{ij})\,] + W_0^4\,(\Sigma_i X_i G_{ji})\,(\Sigma_j X_j G_{ij})$$
$$\{[\alpha_2(\lambda-1)V^{\lambda-4} + \alpha_4 V^{2\lambda-4} + \alpha_4(\lambda-1)V(\lambda)V^{\lambda-4} + \alpha_5(\lambda-1)V^{\lambda-4}E(\lambda)\,] - [\alpha_2 +$$
$$\alpha_4 V(\lambda) + \alpha_5 E(\lambda)\,]\,V^{\lambda-1}V^{-3}\} + W_0^2[\alpha_2 + \alpha_4 V(\lambda) + \alpha_5 E(\lambda)\,]$$
$$V^{\lambda-2}\,G_{ij} \qquad (4.29)$$

Similarly, by substituting equations (4.22), (4.23), (4.24), (4.25), (4.26) into equation (4.15) , one can obtain the effect of changes in r_r and Gij on the Xi's.

$$\delta^2 L/\delta X_i\,\delta r_r = -W_0\,[\,\alpha_1 + \alpha_3 E(\lambda) + \alpha_5 V(\lambda)]\,E^{\lambda-1}\,\delta ir - W_0^2 X_r r_i\,[\alpha_1(\lambda-1)E^{\lambda-2} + \alpha_3(\lambda-1)$$
$$E^{\lambda-2}E(\lambda) + \alpha_5(\lambda-1)E^{\lambda-2}V(\lambda)] - X_r W_0^3(\alpha_5 E^{\lambda-1}V^{\lambda-2}\Sigma_j X_j G_{ij}) \qquad (4.30)$$

The same procedure is applied to find the effect of a change in Gij on the holdings of any asset i, i = 1,......, m+1.

$$
\begin{aligned}
\delta^2 L / \delta X_i \delta G_{rf} = {} & \alpha_1 W_0^2 r_i r_j (\lambda-1) E^{\lambda-2} \delta X_j / \delta G_{rf} + \alpha_2 W_0^2 [1/2(\lambda-2) V^{\lambda-4} W_0^2 (\delta X_i / \delta G_{rf} \\
& \Sigma_j X_j G_{ij} + \delta X_j / \delta G_{rf} \Sigma_j X_i G_{ji} + 2 X_r X_f).(\Sigma_j X_j G_{ij}) + V^{\lambda-2}(\delta X_j / \delta G_{rf} G_{ij} + X_r + X_f)] \\
& + \alpha_3 W_0^3 r_i r_j (E^{2\lambda-2} + E(\lambda)(\lambda-1) E^{\lambda-2}) \delta X_j / \delta G_{rf} + \alpha_4 W_0^2 [1/2 V^{2\lambda-4} \\
& W_0^2 (2 X_r X_f + \delta X_i / \delta G_{rf} \Sigma_j X_j G_{ij} + \delta X_j / \delta G_{rf} \Sigma_j X_i G_{ji})(\Sigma_j X_j G_{ij})] + V(\lambda) 1/2(\lambda-2) \\
& V^{\lambda-4} W_0^2 (2 X_r X_f + \delta X_i / \delta G_{rf} \Sigma_j X_j G_{ij} + \delta X_j / \delta G_{rf} \Sigma X_i G_{ji})(\Sigma_j X_j G_{ij}) + V(\lambda) V^{\lambda-2} \\
& \delta X_j / \delta G_{rf} G_{ij} + X_r + X_f) + \alpha_5 W_0^2 r_i [(\lambda-1) E^{\lambda-2} V(\lambda) (\delta X_j / \delta G_{rf} r_j) + 1/2 E^{\lambda-1} \\
& V^{\lambda-2} W_0 (2 X_r X_f + \delta X_i / \delta G_{rf} \Sigma_j X_j G_{ij} + \delta X_j / \delta G_{rf} \Sigma_j X_i G_{ji}) + \alpha_5 W_0^2 [1/2(\lambda-2) \\
& V^{\lambda-4} E(\lambda) W_0^2 (2 X_r X_f + \delta X_i / \delta G_{rf} \Sigma_j X_j G_{ij} + \delta X_j / \delta G_{rf} \Sigma X_i G_{ji}) (\Sigma X_i G_{ji}) + V^{\lambda-2} \\
& E^{\lambda-1}. W_0 \delta X_j / \delta G_{rf} r_j (\Sigma_j X_j G_{ij}) + V^{\lambda-2} E(\lambda) (\delta X_j / \delta G_{rf} G_{ij} + X_r + X_f)] - \delta\gamma / \delta G_{rf} \\
& = 0 \qquad (4.31)
\end{aligned}
$$

Thus, by estimating equation (4.30) and (4.31), we are able to obtain the yield signs of the various substitution elasticities.

4.6 Summary

In this chapter, the desirability of using the mean-variance utility model was discussed. In order for the investor to obtain an optimal portfolio selection, either a quadratic utility function should be used or the investment outcomes should be normally distributed within the two-moment utility function. However, one can not discriminate against the utility function on theoretical grounds. Therefore, the Box-Cox transformation function is used to carry out the parametric tests for the best utility function. Then, using the share equation in (4.28a), one could estimate the parameters which will then be used for calculating marginal utilities and elasticities of substitution. This procedure will be discussed in the following chapter.

Endnotes

1. The life insurance policies may be classified either as "non-participating" or "participating". Non-participating policies are those which definitely guarantee the premium and the sum insured and do not entitle the insured to receive any benefits other than those expressly stated in the contract. Participating policies, on the contrary, usually require the payment of a premium considerably larger than necessary to meet the company's liability under the contract, and as a consequence, the insured is allowed from time to time to receive a portion of the surplus earnings of the company.

2. Jones, D. L., "Investment Policies of Life Insurance Companies," Harvard University Boston (1968).

3. Markowitz, H., "Portfolio Selection," <u>Journal of Finance</u> (1952).

4. Pyle, D. H., " On the Theory of Financial Intermediation," <u>Journal of Finance</u> (1971).

5. Stowe, D. J., " Life Insurance Company Portfolio Behavior," <u>Journal of Risk and Insurance</u> (1978).

6. More detail about such controversial issues will discuss in the following section.

7. Roy, A., "Safety First and the Holding of Assets," <u>Econometrica</u> (1952).

8. Pyle, D. H., & Turnovsky, S. J., "Safety-First and Expected Utility Maximization in Mean-Standard Deviation Portfolio Analysis," <u>The review of Economics and Statistics</u> (1970).

9. Stowe, D. J., " Life Insurance Company Portfolio Behavior," <u>Journal of Risk and Insurance</u> (1978).

10. According to Arrow, k.,(1964), $d[-u''(x)/u'(x)]/dy$,<=0 i.e. marginal absolute risk-aversion should decrease with an increase in wealth.

11. Levy, H., and Sarnat, M., " Portfolio and Investment Selection," Prentice-Hall International, Inc. (1984).

12. Equation (4.7) holds for each and every period of time and thus the t subscripts are dropped.

13. For further details on the properties of the transformation, see Box, G.E.P., and Cox, D.R. (1964) and Zarembka, P., (1974).

APPENDIX 4

Maximize U (E , V)

$$\text{subject to } \sum_{i=1}^{m+1} X_i = 1 \quad \text{For } X_i >= 0$$

where

$$E = W_0 [\, 1 + \sum_{i=1}^{m+1} X_i r_i \,]$$

$$V = [\, W_0^2 \sum_{i=1}^{m+1} \sum_{j=1}^{m+1} X_i X_j G_{ij} \,]^{1/2}$$

Assumed that two asset are available and applied the Lagrangian method to formulate the following equation.

$$L = U[E\{W_0(1+X_1 r_1 + X_2 r_2)\}, V\{W_0^2 (X_1^2 G_{11} + 2X_1X_2G_{12} + X_2^2 G_{22})\}^{1/2}] + (1 - X_1 - X_2)\,\lambda$$

Appendix 4A

First Order Condition:

$$\delta L/\delta X_1 = W_0 r_1 U_E + 1/2\, V^{-1} U_V W_0^2 \,(2X_1G_{11} + 2X_2G_{12}) - \lambda$$

$$= W_0 r_1 U_E + W_0^2 U_V V^{-1} \,(X_1G_{11} + X_2G_{12}) - \lambda$$

$$\delta L/\delta X_2 = W_0 r_2 U_E + 1/2\, V^{-1} U_V W_0^2 \,(2X_1G_{12} + 2X_2G_{22}) - \lambda$$

$$= W_0 r_2 U_E + W_0^2 U_V V^{-1} \,(X_1G_{12} + X_2G_{22}) - \lambda$$

Generalize the solution

$$\delta L/\delta X_i = W_0 r_i U_E + W_0^2 U_V V^{-1} \sum_j X_j G_{ij} = \lambda \qquad (4.11)$$

Appendix 4.B

$$D = \begin{bmatrix} Z11 & \cdots\cdots & Z1,m+1 & 1 \\ . & & . & . \\ . & & . & . \\ Zj,1 & \cdots\cdots & Zj,m+1 & 1 \\ . & & . & . \\ Zm+1,1 & \cdots\cdots & Zm+1,m+1 & 1 \\ . & & & \\ 1 & \cdots\cdots & 1 & 0 \end{bmatrix}$$

The elements Z_{ij} are calculated by differentiating equations (4.11) and (4.12) again with respect to X_i and X_j which is given below.

Second Order Condition

$$\delta^2 L/\delta X_i \delta X_j = Z_{ij} = W_0^{\,2} r_i r_j U_{EE} + W_0^{\,3} U_{EV} V^{-1} \left[r_i \sum_{i=1}^{m+1} X_i G_{ij} + r_j \sum_{i=1}^{m+1} X_j G_{ij} \right]$$

$$+ W_0^{\,4} (V^{-2} U_{VV} - V^{-3} U_V) \left(\sum_{i=1}^{m+1} X_i G_{ji} + \sum_{i=1}^{m+1} X_j G_{ij} \right) + W_0^{\,2} U_V V^{-1} G_{ij}$$

Appendix 4.C

$$D = \begin{bmatrix} Z11 & \cdots & Z1,m+1 & 1 \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ Zj,1 & \cdots & Zj,m+1 & 1 \\ \cdot & & \cdot & \cdot \\ Zm+1,1 & \cdots & Zm+1,m+1 & 1 \\ \cdot & & & \\ 1 & \cdots & 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{\delta X1}{\delta rj} \\ \\ \frac{\delta Xj}{\delta rj} \\ \\ \frac{\delta Xm+1}{\delta rj} \\ \frac{\delta \lambda}{\delta rj} \end{bmatrix} = \begin{bmatrix} T1 \\ \cdot \\ \cdot \\ \cdot \\ Tj \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ Tm+1 \\ \cdot \\ 0 \end{bmatrix}$$

$$T_i = \delta^2 L/\delta X_i \delta r_j = -W_0 U_E \delta_{ir} - X_r[W_0^2 U_{EE} r_i + W_0^3 V^{-1} U_{EV} \Sigma X_j G_{ij}]$$

where $\delta_{ir} = 1$ for $i = r$

$= 0$ for $i \neq r$

Since $\delta X_k/\delta r_r = \sum_{i=1}^{m+1} T_i D_{rk}/D$

$\delta X_k/\delta r_r$ can be solved as follow:

$$\delta X_k/\delta r_r = -W_0 U_E D_{rk}/D - X_r[W_0^2 U_{EE} \sum_{i=1}^{m+1} r_i D_{ik}/D + V^{-1} W_0^3 U_{EV} \Sigma_i \Sigma_j X_j G_{ij} D_{ik}/D] \quad (4.13)$$

Chapter 5

Estimation And Empirical Results

5.1 Introduction

As we have seen, the theory presented in the previous chapter offers a framework for the theoretical aspect of the insurers' utility function. Since a life insurer's utility function is unknown, we are not able to choose the functional form based on theoretical or econometric grounds. Hence, the Box-Cox (1964) transformation function is employed to provide a variety of new possible functional forms, and parametric tests are carried out to discriminate among the translog, generalized Leontief and square rooted quadratic functional forms. This will be the main focus of the study in this chapter. In addition, an analysis is made of the life insurers' responsiveness toward the yield variation among the several broad types of assets and liabilities.

Generally, there are two approaches to conducting the empirical analysis. One is to conduct a "macro" analysis through the use of aggregate time series data and to draw industry-wise conclusions. The other approach is to adopt the "micro" approach by narrowing the scope of analysis to the individual insurer's investment behaviour and make comparisons between the investment behaviour patterns among life insurers. Although the use of both the macro and micro strategies to capture the whole insurance industry investment behaviour is preferable, the application of macro-type analysis also means that restrictions (like symmetry, linear homogeneity) have to be imposed; also, assumptions (like investment attitudes, environmental factors) need to be made in order to justify the aggregation over individual companies. The model that was discussed in the previous chapter is derived from a micro-theoretical framework where an individual is considered to be the decision unit. As mentioned by Krinsky (1983) :[1]

> " . . . this model is implemented using data for individual life insurance companies rather than data for the entire industry." (pg. 98)

Therefore, instead of making assumptions about symmetry and linear homogeneity to justify the aggregation over individuals, we will adopt the "micro" type of approach to evaluate and compare the different investment attitudes among the life insurers.

This chapter is divided into five sections. The data and sample collection, and the estimation methodology are discussed in section 5.2. Following estimation of the parameters of the demand equations, the best utility function can be chosen and its legitimacy verified. This procedures is discussed in section 5.3. The empirical results are presented in section 5.4. together with an explanation of the portfolio preferences of the life insurance companies. A brief summary and conclusion is given in the last section.

5.2 Data and Sample Selection

In order to evaluate the behaviour of life insurers, a sample of the 8 largest U.S. life insurers was taken. Complete annual data for these samples was obtained from the Moody's Bank and Finance Manual for the United States. Information on the eight largest insurance companies, ranked according to their admitted assets was used for the period 1953 - 1983. For 1983, the total asset value of the sample companies amounted to 389.6 billion dollars, which was about

60% of the total admitted asset values of the whole life insurance industry.

The life insurance companies included in the sample are :

1. Prudential Insurance Company,
2. Metropolitan Life Insurance Company, (Metro)
3. Equitable Life Insurance,
4. Aetna Life Insurance Company,
5. New York Life Insurance Company,
6. John Hancock Mutual Life Insurance,
7. Connecticut General Life Insurance, (Conn)
8. Massachusetts Mutual Life Insurance Company, (Mass).

The financial holding of the life insurers are classified under four assets categories : (a) bonds, (b) stocks, (c) mortgage loans, (d) real estate; and one liability category, net actuarial reserves (NAR).

The figures presented by the Moody's Bank and Finance Manual for assets and liabilities are brokendown in a way which is not consistent with the above noted classification. It is necessary, therefore, to compile the figures for NAR, Bonds and Stocks for the eight life insurance companies for the past thirty-one years in order to proceed with the estimation procedures. The Net Actuarial Reserves include reserves for contracts in force plus claims under consideration, plus deposits, plus provisions for profit to policy-holders, plus other liabilities, less policy loans, less cash, less investment income due and accrued, less outstanding insurance premiums and annuity consideration, less other assets. In this particular case, the policy loans were transfered from the asset side to the liability side for discretionary purposes. This is to ensure that the net actuarial reserves calculated are sufficient for the company to fulfil its contractual obligations. It is impossible to obtain a consistent pattern for different kinds of bonds and stocks because the classification of bonds or stocks varies among the insurance companies. As a consequence, both the bonds and stocks categories are taken as a whole sum.

Besides the asset-holdings, the yield returns were also needed. The annual returns on the bonds were calculated using a weighted average of past annual yields on 5-10 year corporate bonds, high-grade Municipal bonds, State and Local government bonds and the long term Federal government bonds. The computation of the stock returns was based on the Standard and Poor's (S&P) 500 stock index. Conventional mortgage rates and Federal Home Administration (FHA) mortgages were used in calculating the annual return on mortgage loans and real estate.

The rate of return on the net actuarial reserves is dependent upon the life insurer's underwriting profits. As the life insurance policies were designed on a long term basis, any estimation of underwriting profits must include the assumption of future interest rates, mortality rates and expenses. It is assumed that the change in actuarial reserves provides a good estimation of the future claims and expenses. The underwriting profit could therefore be estimated as:

Underwriting profit = revenues - costs[2]

where revenues = premiums + annuity payments collected.

costs = claims paid + change in actuarial reserves + taxes + licences and fees + commissions and general expenses + policy dividends.

Because of the high initial cost of issuing the insurance policies, the expenses of life insurance companies are usually high in the first year and fall gradually in the following years. In order to have an even distribution of expenditure, the first year expenses are spread over a ten year period which is the average length of a policy.

5.2.1 Methodology

First of all, it is necessary to estimate the life insurer's expectations of the future rates of return in order to use them for regression analysis. In reality, the life insurer's expectation of future rates of return will be based on a combination of objective and subjective information. Since subjective information is not available, the parameters can only be estimated through use of the historical time series data for the period 1953-1983. The first ten years of yield data (1953-1962) were employed to calculate the mean returns and variances for each asset/liability as well as sample covariances between asset yields. These sample estimates were then used to calculate the expected return and variance of the portfolio held by each of the eight companies at the end of 1963. The sample of the means, variances and covariances for 1964 were calculated by dropping the 1953 data and adding the data for 1964. These new estimates together with the asset proportions held by the insurance companies at the end of 1964 provide the 1964 portfolio mean, variance and covariance. Thus, the same procedure of adding the 1965 data point and dropping the 1954 data point and so on has been used. By employing this rolling-sample technique, a 21 data point was generated and utilized to estimate the utility function parameters (see Appendix 5a).

5.2.2 Estimation of the demand equations

In order to estimate the coefficients of the demand equation (4.28a) from chapter 4, a nonlinear estimation method is required. The best-known method of estimation dealing with nonlinearities is the maximum likelihood method. Maximum-likelihood (ML) estimators are consistent and asymptotically efficient. Besides, the ML method can be linked up with the likelihood ratio test to verify overall hypotheses about the system. The computational burden for Maximum-likelihood is roughly the same as for the three-stage least squares in the case of non-linear models.

The estimated system is

$$X = K \, . \, Z^{-1} r^{*} + v$$

The column vector of disturbances at time t is defined as :

$$v(t) = \{ v_1(t), v_2(t), \ldots\ldots\ldots, v_{m+1}(t) \} \qquad t=1, \ldots\ldots\ldots T.$$

and the associated disturbance (assumed constant) variance-covariance matrix is represented as Ω.

The purpose of appending disturbance terms to the budget share equations is to provide a stochastic specification for estimating the demand system. The share equations are assumed to be stochastic because of errors in optimization. Since the asset proportions must sum to unity , the m+1 components in v(t) add up to zero at each annual observation and Ω in each of the models is singular and nondiagonal. In this case, the density of v(t) is not defined since only m-1 share equations are independent of each other for any given value of λ. Thus, if the estimation procedure is to be efficient, one of the equations must be dropped in each of our models and the resulting vector will have a non-singular distribution. As Barten (1969) pointed out :[3]

> ". . . for the purpose of maximization of the likelihood function it is completely irrelevant what component is deleted or, equivalently, what equation is dropped from the system" (p.25).

The same procedure of dropping one equation has been followed by various researchers such as Darroug and Diewert (1977), Applebaum(1979), Berndt and Khaled(1979), Aivazian, Callen, Krinsky, and Kwan (1983). The last estimated demand equation is therefore dropped in order to obtain a complete vector of disturbance terms and the complete covariance matrix Ω.

5.3 Choosing the Best Utility Function

Despite the fact that the theoretical discussion has pointed out that the quadratic utility function is the best functional forms to employ, it is not sufficient to determine such a utility function based on econometric grounds. Therefore, all the functional forms will be estimated including the unrestricted system where λ is a free parameter. The purpose of estimating the unrestricted parameter is to use its log likelihood ratio as a yardstick to test the eligibility of the other functional forms. As the demand equations are homogeneous of degree zero in the α_i parameters, we need a normalization on the parameters in order to identify the parameters. We therefore chose to normalize the remaining parameters with respect to α_5.[4]

Four different budget share models were estimated; the translog (λ=0), the generalized Leontief (λ=1/2), the square root quadratic (λ=1) and the unrestricted system where λ is a free parameter. The results are presented in Table 5.1.

In order to choose the utility function that best fits the data, one has to use the test statistics (-2lnL)[5] to determine whether the value is within the Chi-Square critical range. Table 5.2 contains the test statistics (-2lnL) for all the companies in the sample. For instance, for Equitable Life, the result are : 17.152 (translog), 63.770 (GL), 5.404 (Q)[6], while the Chi-square critical value is 6.635 at 1% level. This implies that we cannot reject the quadratic utility function at the 1% significance level.

The overall results show that:

(1) For all the eight companies, we can not reject the quadratic utility function at the 0.5%[7] significance level and for seven out of eight companies at the 1% significance level.

(2) The Translog specification cannot be rejected since five out of eight companies are within the 0.5% and 1% significance level.

(3) Unfortunately, none of the generalized Leontief function falls within the 0.5% or 1% limit.

Thus, from the above testing results, one could infer that the quadratic utility function is the best representation of the preferences of United States life insurance companies.

In addition, it is necessary to utilize the theory of asset demand to examine the validity of the chosen utility form. From the theory of asset demand, it was expected that the "optimal" utility function to satisfy the following conditions:[8]

(1) the sign of U_E/U_V should be negative.

(2) the own elasticity with respect to expected return should be positive for all assets and negative for liabilities.

(3) the own elasticity with respect to variance (risk) should be negative for all assets and liabilities.

TABLE 5.1

PARAMETER ESTIMATES FOR THE FOUR FUNCTIONAL FORMS

Functional Form	Pruden-tial	Metro Life	Equitable Life	Aetna Life	New York Life	John Han-cock Life	Conn Life	Mass Life
Translog ($\lambda = 0$)								
$\alpha1/\alpha5$	73.755	175.574	81.476	-120.219	-6.465	41.478	-5335.97	18.185
$\alpha2/\alpha5$	- 3.854	- 2.131	- 1.976	13.216	-74 .341	- 1.117	- 631.66	1.765
$\alpha3/\alpha5$	-19.219	- 47.407	- 25.572	50.240	4.424	- 14.317	2731.79	14.442
$\alpha4/\alpha5$	-.5886 E-01	.3522	.2038	1.858	.3646	.2849	-79.892	.6424
Log likelihood function	-188.052	229.223	208.993	147.979	233.414	223.562	186.280	227.254
Generalized Leontief ($\lambda = 1/2$)								
$\alpha1/\alpha5$	7.493	12.167	11.310	7.847	.1541E-07	7.489	.1228E-07	6010.21
$\alpha2/\alpha5$	-.5705	- 1.042	-1.391	-1.557	-301.998	-.5701	-537.001	-2556.73
$\alpha3/\alpha5$	-1.199	-19.023	-6.008	-2.640	-.2388 E-07	-1.197	-.1829E-07	-8105.46
$\alpha4/\alpha5$	5.119	7.327	2.158	1.205	172.031	5.118	-158.415	-1040.45
Log likelihood function	232.730	295.277	232.302	236.740	285.605	236.048	229.693	281.559

TABLE 5.1 (CONTINUED)

PARAMETER ESTIMATES FOR THE FOUR FUNCTIONAL FORMS

Functional Form	Prudential	Metro Life	Equitable Life	Aetna Life	New York Life	John Hancock Life	Conn Life	Mass Life
Square root quadratic ($\lambda = 1$)								
$\alpha 1/\alpha 5$	3.634	5.527	-776.7	3.368	.9575	1.891	1376.5	788.887
$\alpha 2/\alpha 5$	- 6.033	-6.055	48.13	-2.650	-10.704	-5.854	298.287	-1088.31
$\alpha 3/\alpha 5$	-.07915	-.1302	41.99	-.3165	.002556	-.08047	-349.337	-204.892
$\alpha 4/\alpha 5$	.1970	.1909	2.362	.4757	.9141	.5856	-88.279	-293.037
Log likelihood function	208.649	223.046	205.821	225.410	226.395	227.771	178.018	229.153
Unrestricted								
λ	.21349	.3879	-.3133	-.6851E-01	.3748	-.1351	-.2402	-.5819
$\alpha 1/\alpha 5$	216.643	13.097	22.635	362.719	5.807	7.384	645.635	40.814
$\alpha 2/\alpha 5$	296.280	127.396	-.3491	31.299	-5.590	3.321	-26.589	.5789
$\alpha 3/\alpha 5$	10.942	13.097	-35.363	3284.32	9.473	1.515	1197.51	50.078
$\alpha 4/\alpha 5$	15.323	118.411	.01618	6.056	-4.854	.1568	-1.438	.002067
Log likelihood function	208.633	227.358	200.417	228.805	232.367	222.223	184.717	224.732

TABLE 5.2

LIKELIHOOD RATIO TEST RESULTS FOR THREE FUNCTIONAL FORMS

Insurer	Test Statistic (-2lnL)			
	Translog	Generalized Leontief	Quadratic	Chi-Square Value(1%)
Prudential	41.162	48.194	.016*	6.635
Metro	3.730*	135.838	4.312*	6.635
Equitable	17.152	63.770	5.404*	6.635
Aetna	161.652	15.870	3.395*	6.635
New York	2.094*	106.476	5.972*	6.635
John Hancock	2.678*	27.650	5.548*	6.635
Conn	3.126*	89.952	6.699	6.635
Mass	5.044*	113.654	4.421*	6.635
Results	5/8	non	7/8	

* Value which is within the Chi-square limit

(4) the principal minors of the bordered Hessian [e.g., chapter 4, equation (4.13)] should alternate in sign.

Table 5.3 lists the sign of U_E/U_V for 1983 derived from the best utility function. Since the rates varied little over the sample period, only the results for 1983 are reported to represent the entire period of 1963-1983. In six out of eight companies U_E and U_V were opposite in sign, and thus in line with the theory. New York and Massachusetts life are the two companies for which show the negativity condition is violated.[9]

Since the majority of the cases showed that the quadratic is both consistent with the data and the mean-variance portfolio theory, it was concluded that it represents the life insurer's investment preference well.

TABLE 5.3

THE SIGN OF U_E/U_V FOR THE INSURERS

Insurer	Sign
Prudential	< 0
Metro	< 0
Equitable	< 0
Aetna	< 0
New York	> 0
John Hancock	< 0
Conn	< 0
Mass	> 0

5.4 The Elasticities and their implications

The elasticities of with respect to expected returns and variances were estimated using equations (4.13) and (4.15). The estimation of the elasticities of demand with respect to expected returns and variances depends very much on the sign of U_E and U_V. Equation (4.13) and (4.15) can be decomposed into two effects. Aivazian (1976) identified the last term in each equations as the average productivity effect, while the first term on the right is the pure marginal productivity effect. As the marginal productivity effects are larger in magnitude than the average productivity effects, the signs of $\delta X_k/\delta r_r$ and $\delta X_k/\delta G_r^2$ are therefore determined by the first term of the equations. In other words, if U_E or U_V is negative, the sign of $\delta X_k/\delta r_r$ and $\delta X_k/\delta G_r^2$ will appear to be positive and vice versa.

Table 5.4 (a) and (b) consist of the own elasticities of with respect to expected return and variance respectively. For the six companies with opposite sign for U_E and U_V, all own elasticities of substitution are positive with respect to expected return, and negative with respect to the variance. Therefore, the pure marginal productivity effect in the case of expected return is unambiguously positive and in the case of risk unambiguously negative.

The row which contains the own interest elasticities for net actuarial reserve in table 5.4(a), shows the elasticity is negative for five out of eight companies. One explanation for this is that when the expected costs of underwriting insurance goes up, "ceteris paribus", the expected profit of the insurance company will be reduced. Therefore, in order to avoid any losses, the insurance company may reduce or limit their sales of the policy. As a result, the proportion of

NAR it holds on its balance sheet will also be reduced. Generally, the own elasticities indicate that a one percent change in expected return has a larger impact on real estate and mortgage than on bonds or stocks. This is intuitively plausible given the unstable nature of stocks and bonds. It is unlikely that a one percent change in expected returns will induce a change in demand for stocks and bonds as compared to the mortgage or real estate.

All of the companies show the same negative sign for the own elasticities with respect to variance (see Table 5.4b). This implies that an increase in the variance (risk) of the expected return of an asset will lead to a reduction in its proportion held in the balance sheet of the company. The elasticities in terms of the variances are smaller in magnitude for bonds and stocks in comparison with mortgages and real estate. These results imply that life insurance companies are facing higher risk when they invest in bonds and stocks. Again, the results demonstrate that unless the expected rate of return is increased by a fairly large amount, the life insurer will prefer to invest in stable income securities than to face uncertainty. Stocks are smaller in magnitude in terms of variance which suggests that the holding of stocks is less risky than bonds, mortgages or real estate.

The own variance elasticities of NAR are notably small for all companies. Even though it was mentioned earlier that if the expected cost of underwriting is increasing, the life insurance company will reduce or limit the amount of their sales. However, these small variance elasticities suggested that because of the competitive environment, the life insurance company still need to underwrite insurance in order to stay in business. The magnitude of the variance elasticities are generally larger than the own elasticities with respect to expected return. One of the plausible explanations might be that any change in one asset's expected return on variance will have an effect on its correlation with other assets and thus affect the optimal portfolio allocation.

We now proceed to look at the cross elasticity signs in Tables 5.5 and 5.6, and to analyze the implications of the signs. Theoretically, the off diagonal elasticities with respect to expected returns or variances can be of any sign. This is, because of the many variables influencing them, one could hardly predict any of these signs. Generally, the signs listed in table 5.5 do not show any clear-cut pattern of substitutability among the assets. Nevertheless, the majority of the signs do show that real estate and mortgages are positively related. This means that real estate and mortgages are complementary to each other. Similarly, bonds and stocks also appear to have a complementary effect on each other. On the other hand, there is a fairly strong degree of substitutability between real estate and stocks. The magnitude is especially significant for the firm of John Hancock. Generally, mortgages are a weak substitute for bonds and stocks.

TABLE 5.4 (a)

OWN-ELASTICITIES

Own Expected Return Elasticity	Pruden-tial	Metro Life	Equitable Life	Aetna Life	New York Life	John Han-cock Life	Conn Life	Mass Life
B	1.0373	2.1613	.2346	.2693	-.23263	36.8719	.16656	-.8140
S	5.7388	3.4590	.4219	1.83901	- .25961	3.69898	6.13806	-12.0716
M	19.814	5.6074	.3582	5.0390	-.36055	40.6575	9.031	-14.3077
RE	7.4038	75.1775	.4378	70.3015	-.40509	23.7203	6.5452	-17.2826
NR	.19439	-.31477	.3651E-01	-.656E-01	-.416E-01	8.2778	-.1920	- .30837

TABLE 5.4(b)

Own-Elasticity
with respect to
Variance

B	-17.1001	-10.6544	-3.9760	-11.556	- 5.3056	-15.2351	-3.2381	-7.6397
S	-10.8676	- 2.8317	-2.6012	-8.8837	-3.9794	-14.6866	-2.6182	-6.5010
M	-27.5948	-77.6303	-4.0885	-11.729	-5.4063	-19.6159	-3.2800	-8.8077
RE	-27.3814	-76.9565	-4.0831	-117.449	-5.3290	-19.4735	-3.3154	-8.7034
NR	-.205208	-.757891	-.47824	-.59807	-.42533	-1.49543	-.21663	-.69198

TABLE 5.5

THE CROSS-ELASTICITIES WITH RESPECT TO EXPECTED RETURN FOR THE END OF 1983

Prudential

	B	S	M	RE	NR
B	+1.0373	+.22766	-.18330	-.16403	-.65773
S	+1.7977	+ 5.7388	-1.9981	-1.8064	-.81462
M	-.24660	-.34043	+19.814	+.25158	-.95569E-01
RE	-1.0702	-1.4924	+1.2199	+7.4038	-.41984
NR	-.19962	-.31309	+.21560	-.19531	+.19439

Metro

	B	S	M	RE	NR
B	+2.1613	-.83026	+.51905	-.57219	+.81167E-01
S	+9.8288	+3.4590	-12.066	-13.350	+1.94649
M	-.76487	+1.4944	+5.6074	+1.0534	-.14669
RE	-9.7584	-19.2334	+12.1924	+75.1775	-1.8706
NR	+.29249	-.59251	- .35872	-.39527	-.31477

Equitable

	B	S	M	RE	NR
B	+.2346	+.7865E-02	-.8055E-01	-.1474E-01	+.1057E-01
S	+ 2.4546	+.42190	- 3.5431	-.65307	-.95090
M	+.5004E-01	-.7075E-02	+.35828	+.1347E-01	-.7992E-02
RE	-.35023	- .4969E-01	-.51484	+.43782	+.6380E-01
NR	.1206E-02	-.3475E-03	+.1467E-02	+.3064E-03	+.3651E-01

TABLE 5.5 (CONTINUED)

Aetna

	B	S	M	RE	NR
B	+.2693	+.13991	+.19089E-02	-.9488E-01	-.5490E-01
S	- 2.4735	+1.83901	+.11061	-5.555	-3.8724
M	-.24537	-.8042E-01	+5.0390	+.560E-01	-.3422E-01
RE	-.44970	-1.4892	+.2062E-01	+70.3015	-.63257
NR	- .19188	-.69619	-.8463E-02	+.42423	-.656E-01

New York

	B	S	M	RE	NR
B	-.2326	-.4878E-02	+.3632E-01	+.3598E-02	+.8412E-02
S	+.96182	-.25961	+.90312	- .9035E-01	-.23447
M	+.5483E-01	+.6915E-02	-.36055	- .5232E-02	-.1177E-01
RE	+.64683	-.8238E-01	+.62301	-.40509	-.14065
NR	-.6822E-01	+.9645E-02	+.6327E-01	+.6345E-02	-.416E-01

John Hancock

	B	S	M	RE	NR
B	+36.8719	+9.8996	+ 6.9281	+7.070	- 3.520
S	+100.698	+3.69898	-116.186	-119.771	-66.948
M	-4.9871	-8.2223	+40.6575	+6.01989	-2.8397
RE	- 29.7167	-494.917	+3.5150	+23.7203	-10.2734
NR	-5.3745	10.0494	6.0234	6.1996	+8.2778

TABLE 5.5 (CONTINUED)

	Conn				
	B	S	M	RE	NR
B	+.16656	+.9495E-01	-.1316E-01	- .6494E-01	-.5976E-01
S	+ 5.6134	+6.13806	- 3.1045	- 15.4917	13.8023
M	-.2012E-01	-.803E-01	+9.031	-.5653E-01	-.4245E-01
RE	-.30575	-1.2338	+.17405	+ 6.5452	- .65487
NR	+.49207E-01	- .22658	-.2688E-01	.13498	-.19207

	Mass				
	B	S	M	RE	NR
B	-.81407	-.15910	+.14269	+.11168	-.7638E-01
S	-4.5997	-12.0716	-4.9851	+3.9421	- 2.90499
M	+.18201	-.22068	-14.3077	+.15905	+ .10542
RE	+2.9370	+3.5865	- 3.2690	-17.282	-1.7093
NR	+.20759	-.27237	+ .22329	+.17615	-.30837

TABLE 56

THE CROSS-ELASTICITIES WITH RESPECT TO VARIANCE FOR THE END OF 1983

Prudential

	B	S	M	RE	NR
B	-17.1001	+.82783	+2.5527	+.60663	- 6.9432
S	-46.904	-10.8676	-27.8273	+ 6.6807	-85.9921
M	-6.4422	-1.2378	-27.5948	-.93044	-10.088
RE	+27.955	-5.4268	+16.990	-27.3814	-44.3192
NR	- 5.2148	-1.1384	-3.0026	-.72234	-.205208

Metro

	B	S	M	RE	NR
B	-10.6544	+1.4401	+7.1858	+.58573	-19.5432
S	- 344.045	-2.8317	-166.212	-13.6665	-468.667
M	+.76487	+ 1.4944	-77.6303	+1.0534	-.14669
RE	+ 9.7584	+19.2334	+12.1924	-76.9565	-1.8706
NR	-.29249	-.59251	-.35872	-.39527	-.757891

Equitable

	B	S	M	RE	NR
B	- 3.9760	+ .4849E-01	-.91923	- .13755	- .13850
S	- 41.5842	-2.6012	-40.4318	-6.0904	+12.455
M	- . 84785	-.4348E-01	-4.0885	+.12562	-.10468
RE	- 5.9334	+.30637	+ 5.8750	-4.0831	+.83573
NR	-. 2043E-01	+.2142E-02	+.1074E-01	+.2858E-02	-.47824

TABLE 5.6 (CONTINUED)

Aetna	B	S	M	RE	NR
B	-11.5560	+.6758E-01	+2.7853	-.15852	- .75162
S	+106.134	-8.8837	-161.390	-9.2813	-53.014
M	-10.5285	-.3884E-01	-11.7297	-.9355E-01	-.468544
RE	+19.2957	-.71942	-30.091	-117.449	-8.66007
NR	- 8.2334	-.33631	+12.3480	+.70874	-.59807

New York	B	S	M	RE	NR
B	-5.3056	-.7478E-01	-.54464	-.4733E-01	- .85999
S	+21.936	-3.9794	-13.542	+ 1.1886	-23.9697
M	+ 1.2505	+ .10600	-5.4063	- .6882E-01	-1.2039
RE	-14.7520	-1.2628	- 9.3418	-5.3290	-14.3785
NR	- 1.5560	-.14785	- .94873	+.83477	-.425333

John Hancock	B	S	M	RE	NR
B	-15.2351	+.39306	-3.3425	- .57923	+ 6.3590
S	+ 52.534	-14.6866	+ 56.2559	+ 9.8126	-120.946
M	-2.6016	-.32646	-19.6159	-.49319	+5.1301
RE	+15.5023	-1.9650	+16.9587	-19.4735	-30.8312
NR	+2.8037	-.39900	+ 2.9061	-.50792	-1.49543

TABLE 5.6 (CONTINUED)

	Conn				
	B	S	M	RE	NR
B	-3.2381	-.4050E-02	+.47795	+.3289E-01	-.57257
S	-109.128	-2.6182	+112.750	-7.8473	-155.670
M	- .39122	+.3425E-02	-3.2800	-.2863E-01	-.47888
RE	+ 5.9440	-.52629E-01	- 6.3213	-3.3154	-7.3861
NR	+.95778	-.9665E-02	- .97642	+.6837E-01	-2.1663

	Mass				
	B	S	M	RE	NR
B	-7.6397	+.8567E-01	+ .87842	+.5624E-01	+1.5677
S	+48.8136	-6.5010	-30.6859	+ 1.9852	+ 65.1866
M	+1.9380	-.1843	-8.8077	+8.009E-01	- 2.3655
RE	+ 31.1766	+1.9316	- 20.1243	-8.7034	+38.3566
NR	+ 2.1975	-.14668	-1.3746	-.8871E-01	-.69198

Definitions:

B - Bonds

S - Stocks

M - Mortgages

RE - Real Estate

NR - Net Actuarial Reserves

Finally, an attempt has also been made to analyse the investment attitudes differences of the mutual and stock life insurance companies. The difference between a mutual and stock company is that a mutual company is not allowed to issue any nonparticipating policies while the stock company may issue either nonparticipating or participating policies. As the policy-holder is also a member in the mutual company, it seems reasonable to predict that the participating policy-holders (members) are willing to undertake more risks as compared to the stock-holders in the stock companies. In the sample, two out of the eight companies were stock companies, the Aetna and Connecticut life insurance companies. Because of the small sample constraint, one stock and one mutual company were chosen for comparison purposes. The comparison between Metropolitan and Aetna are shown in Table 5.8

From the table, it can be observed that the own elasticities with respect to variances are larger in magnitude for the stock company than the mutual company, except for the mortgage and the net actuarial reserves components. However, when the elasticities with respect to expected return are considered, the mutual company elasticities are larger in magnitude than the stock company. This implies that the mutual company is taking higher risk with higher expected yield returns. Based upon the overall results, it can be said that the Metro (mutual) life company adopted a more aggressive approach in its investment strategy than the Aetna (stock) life company. This analysis may provide some insight into the reason why mutualization was so successful as compared to the more traditional stock company in the period of the early 60's. Similar tests were carried out to compare other stock and mutual companies. Based on the overall results, it can be concluded that stock life companies are more conservative in their portfolio selection than the mutual life companies.

TABLE 5.8

A COMPARISON BETWEEN METROPOLITAN LIFE (MUTUAL) AND AETNA LIFE (STOCK)

Asset	Own Elasticities with Respect to					
	Variance			Expected Return		
	Metro	Aetna	$\frac{(3)=(1)-(2)}{(1)} \times 100$	Metro	Aetna	$\frac{(6)=(4)-(5)}{(4)} \times 100$
	(1)	(2)	(3)	(4)	(5)	(6)
B	-10.6544	-11.556	-8.46%	2.1613	.2693	87.5%
S	-2.8317	-8.8837	-213.7%	3.4590	1.83901	46.8%
M	-77.6303	-11.729	84.9%	5.6074	5.0390	10.2%
RE	-76.9565	-117.449	-52.6%	75.1775	70.3015	6.48%
NAR	-.757891	-.59807	21.1%	-.31477	-.06560	79.2%

5.5 Summary

From the above analysis, a certain degree of insurers' responsiveness toward the expected return and the variance can be detected. However, it is difficult to generalize this conclusion as the analysis is constrained by the small size of the sample. Nevertheless, the assumption that the demand for a security is positively related to its own yield, and negatively related to the variance was verified. There is a certain degree of complementary effect between real estate and mortgages as well as between bonds and stocks. Different investment strategies adopted by the stock and mutual life companies were also observed. Because of the broad category of the assets and limited sample collection, it was not possible to go into a more in-depth analysis.

Endnotes

1. Krinsky, I., "Mean-Variance Utility Functions and the Investment Behaviour of Canadian Life Insurance Companies," Unpublished Ph. D. Dissertation, McMaster University, Hamilton, Canada (1983).

2. A similar approach was used by S. Kellner and G.F. Mathewson (1980).

3. The Barten proof relates only to Full Information Maximum Likelihood (FIML) parameter estimates. Independently, S.Kmenta and R.F. Gilbert (1968) showed that iterated OLS converged to FIML using Monte Carlo techniques and P.Dhrymes (1973) proved this convergence analytically; that is, he proved that iterated Seemingly Unrelated Regression (SUR) is asymptotically equivalent to FIML. It is this SUR that is used in this thesis.

4. Aivazian, Callen, Krinsky, and Kwan (1983) adopted the same procedure, with the backing opinions at Christiensen, Jorgenson, and Lau (1975), Berndt, Darrough, and Diewert (1977), and Appelbaum (1979). Aivazian et. al point out that even with the use of a different normalization, the results are invariant to this normalization.

5. -2lnL is asymptotically distributed $X^2(1)$ where L is the ratio of the value of the unrestricted likelihood function to the value of the restricted likelihood function.

6. Applebaum (1979) pointed out that the square root quadratic and the ordinary quadratic are empirically indistinguishable since d does not appear in the estimating system. The purpose of using the quadratic instead of square root quadratic is to test the consistency of the theory as discussed in chapter 4.

7. The Chi-square value at the 0.5% level is 7.879

8. The specification of these signs is based on the studies done by Barret,Gray and Parkin (1975), Kahane and Nye (1975), and Krinsky (1982). A theoretical discussion of the signs may be found in Beirwag and Grove (1968) or Aivazian (1976).

9. By extending the test to the other utility forms, the same sign appeared on the other three flexible forms for the two companies.

Appendix 5.a

ASSET HOLDING BY LIFE INSURANCE COMPANIES AND THEIR ASSOCIATED EXPECT RETURNS
(1953 - 1983)

PRUDENTIAL LIFE INSURANCE COMPANY

A. Holdings ($)

1.	2.	3.	4.	5.
05153934807	00306508303	04365079350	00235973491	09963223076
05288788740	00322445208	04874403570	00283002548	10616617080
05614816625	00341260481	05245566122	00317545331	11355062540
05791711296	00295678028	05742095341	00350640987	11943703340
05998478616	00302671111	06074441297	00398925069	12577497520
06331253701	00373297345	06289192349	00493565312	13286150520
06714887581	00439966180	06611250473	00565282762	14089241800
07095207914	00424174246	07063077585	00575612286	14891371600
07526079781	00560355433	07366197566	00620671740	15765235030
08036529448	00540228998	07805757741	00635205032	16677313110
08544071271	00616315643	08254636623	00690765910	17709445830
08883661714	00712836044	08910173987	00728814369	18746618870
09358583835	00930109332	09382439959	00761891984	19905162240
09701698402	00912730447	09993134060	00831942271	20855358820
10224424880	01599515394	10331034258	00862180622	13280644360
10641249190	01367139217	10605907226	00948565110	22966333000
10993056731	01288126654	10809044651	00965399279	23444227840
11498593780	01294352464	10988991345	00970752827	24139797360
12008281800	01758803229	11052730892	01049858695	27473031170
12859054763	02505913346	11085743597	01091464884	25899766480
12899507000	02862172000	11652507000	01262689000	26989861000
13041335000	02577023000	12305870000	01446054000	27877766000
14249205000	03214983000	12411159000	01678235000	29753688000
16812097000	03796618000	12314826000	01840809000	32954946000
18884412000	03722689000	12465426000	01873526000	35001335000
20559667000	03937844000	12830039000	02089012000	39790834000
21138825000	04506929000	13907963000	02323562000	39715989000
21154942000	05199577000	14862548000	02680825000	41099355000
21236141000	04462047000	14927819000	03041462000	41253510000
19800147000	04765138000	14674977000	03312133000	40048710000
23591390000	04964613000	13902403000	03336141000	42825583000

B. Expected Returns (percentage point)

1.	2.	3.	4.	5.
0.0282	0.0509	0.0500	0.0415	0.0371
0.0252	0.0450	0.0470	0.0435	0.0461
0.0271	0.0400	0.0490	0.0465	0.0335
0.0326	0.0408	0.0460	0.0518	0.0350
0.0352	0.0434	0.0520	0.0550	0.0247
0.0339	0.0415	0.0540	0.0582	0.0200
0.0396	0.0369	0.0480	0.0610	0.0411
0.0427	0.0395	0.0500	0.0616	0.0377
0.0378	0.0358	0.0510	0.0532	0.0364
0.0384	0.0476	0.0520	0.0540	0.0293
0.0374	0.0357	0.0581	0.0546	0.0285
0.0394	0.0344	0.0580	0.0545	0.0234
0.0402	0.0348	0.0583	0.0547	0.0255
0.0459	0.0399	0.0640	0.0638	0.0247
0.0483	0.0395	0.0653	0.0655	0.0170
0.0555	0.0396	0.0712	0.0721	0.0140
0.0601	0.0430	0.0799	0.0829	0.0136
0.0729	0.0489	0.0852	0.0903	0.0099
0.0608	0.0438	0.0775	0.0770	0.0040
0.0612	0.0419	0.0764	0.0753	0.0053
0.0634	0.0451	0.0830	0.0819	0.0193
0.0505	0.0581	0.0922	0.0955	0.0233
0.0739	0.0554	0.0910	0.0919	0.0088
0.0735	0.0507	0.0899	0.0882	0.0015
0.0686	0.0555	0.0895	0.0868	0.0198
0.0738	0.0619	0.0968	0.0970	0.0241
0.0847	0.0659	0.1115	0.1087	0.0234
0.1113	0.0691	0.1395	0.1344	0.0372
0.1259	0.0749	0.1652	0.1631	0.0027
0.1289	0.0794	0.1579	0.1531	0.0225
0.1019	0.0648	0.1343	0.1311	0.0152

METROPOLITAN LIFE INSURANCE COMPANY

A. Holdings ($)

1.	2.	3.	4.	5.
08437418068	00172718060	02336397135	00443446661	10674587540
08840867978	00166661414	02632679174	00483200825	11342909540
09063287941	00156286584	03169980733	00518255723	12146178390
09163188668	00138634991	03840160293	00536985812	12801028180
09541339000	00134387421	04121771557	00576170553	13439280020
10016802136	00146312762	04324791465	00562775538	14190422850
10592965159	00145584735	04544266609	00571592331	14984372860
10736264498	00189796833	05054339906	00587514682	15664351230
10967674768	00209942619	05529909204	00610031217	16331159650
11299286282	00199034681	05987558451	00601463061	17049324380
11590528170	00200694980	06513096202	00594119200	17889695210
11756997620	00218532053	07243621009	00554941406	18627711020
11974898260	00246139084	08035382213	00573048182	19494220190
12053355390	00245033402	08836417868	00555673334	20246002790
12496384000	00316532887	09311010631	00542005122	21149644190
12982244240	00432958054	09839182702	00493276103	21909684300
13176768170	00446696252	10352374666	00494134637	22511072520
13116885960	00508359290	11088061420	00516721455	23467637000
12992024870	00724921129	11369144978	00606639459	25454329070
14252034860	01350791844	11329661565	00622157853	25112688650
14871621250	01588954805	11658051776	00574380624	26422955800
15466940670	01384277152	11922891693	00559322634	27552394490
17203812410	01523182557	12002874950	00660069527	29402653590
19033262130	01906822741	11829397509	00723780646	22295432430
21433181000	01816793000	11432556000	00753791000	33236453000
23569320000	01877842000	11625315000	00709191000	35271575000
24562075000	02102388000	12829798000	00642652000	37522356000
25533771000	02301737000	14155313000	00631510000	39688261000
26942562000	02238100000	15038421000	00929768000	42018990000
28470671000	02387380000	15326668000	01060894000	44185706000
31289235000	03205340000	15168987000	01242530000	42341511000

B. Expected Returns (percentage point)

1.	2.	3.	4.	5.
0.0282	0.0509	0.0500	0.0415	0.0415
0.0252	0.0450	0.0470	0.0435	0.0435
0.0271	0.0400	0.0490	0.0465	0.0465
0.0326	0.0408	0.0460	0.0518	0.0518
0.0352	0.0434	0.0520	0.0550	0.0550
0.0339	0.0415	0.0540	0.0582	0.0582
0.0396	0.0369	0.0480	0.0610	0.0610
0.0427	0.0395	0.0500	0.0616	0.0616
0.0378	0.0358	0.0510	0.0532	0.0532
0.0384	0.0476	0.0520	0.0540	0.0540
0.0374	0.0357	0.0581	0.0546	0.0546
0.0394	0.0344	0.0580	0.0545	0.0545
0.0402	0.0348	0.0583	0.0547	0.0547
0.0459	0.0399	0.0640	0.0638	0.0638
0.0483	0.0395	0.0653	0.0655	0.0655
0.0555	0.0396	0.0712	0.0721	0.0721
0.0601	0.0430	0.0799	0.0829	0.0829
0.0729	0.0489	0.0852	0.0903	0.0903
0.0608	0.0438	0.0775	0.0770	0.0770
0.0612	0.0419	0.0764	0.0753	0.0753
0.0634	0.0451	0.0830	0.0819	0.0819
0.0505	0.0581	0.0922	0.0955	0.0955
0.0739	0.0554	0.0910	0.0919	0.0919
0.0735	0.0507	0.0899	0.0882	0.0882
0.0686	0.0555	0.0895	0.0868	0.0868
0.0738	0.0619	0.0968	0.0970	0.0970
0.0847	0.0659	0.1115	0.1087	0.1087
0.1113	0.0691	0.1395	0.1344	0.1344
0.1259	0.0749	0.1652	0.1631	0.1631
0.1289	0.0794	0.1579	0.1531	0.1531
0.1019	0.0648	0.1343	0.1311	0.1311

EQUITABLE LIFE ASSURANCE SOCIETY

A. Holdings ($)

1.	2.	3.	4.	5.
04630676513	00133831750	01606034217	00187176217	06251214019
04885305798	00187271911	01818351449	00187644612	06716796616
05023623434	00197219365	02111602982	00211602982	07157607914
05074823909	00176759269	02484607621	00201229656	07514680913
05122362689	00172330357	02814934601	00201619535	07856024902
05211198074	00193094152	03123651059	00187319642	08225142351
05241771583	00215913652	03408884179	00207697146	08638781257
05324602898	00256529532	03582803198	00234491295	08935593291
05403152263	00324003449	03768454540	00257643060	09331699564
05492302367	00352670853	03998933704	00283105908	09511223544
05480992641	00419949418	04324591839	00317278996	10146029720
05459657142	00476387303	04698038005	00345831352	10364654170
05358135731	00548985823	05150657407	00350612351	11297504030
05217943057	00514794382	05514840922	00361902528	10938413910
05229996942	00607539041	05764077195	00372821227	11272982130
05283016900	00657702702	05941556760	00403930033	11555482370
05241218010	00621099336	06061477168	00447749163	11149777610
05140345075	00601594903	06097333060	00474886666	13645807690
05544126974	00744072285	06108087449	00491056983	12257515460
05631394630	00969813643	06211235600	00522449397	12630112920
05735773752	00886019256	06509208286	00530282547	12923810090
05686868909	00741493399	06828306717	00579933894	13265656900
06299963095	00915384539	07291633494	00681195716	14494310780
07421633391	00955687403	07774030419	00765964485	16187973900
08243610000	00903812000	08435563000	00896317000	17715216000
09345607000	00387035000	09229746000	01210615000	19318228000
09738585000	00376432000	09911851000	01327902000	20495461000
09938309000	00442045000	10616726000	01393954000	21330947000
09713111000	00381316000	10613150000	01658327000	21167655000
10636415000	00444513000	10613898000	01722692000	21697113000
11526547000	00621100000	11247074000	01678249000	24908184000

B. Expected Returns (percentage point)

1.	2.	3.	4.	5.
0.0282	0.0509	0.0500	0.0415	0.0385
0.0252	0.0450	0.0470	0.0435	0.0441
0.0271	0.0400	0.0490	0.0465	0.0385
0.0326	0.0408	0.0460	0.0518	0.0336
0.0352	0.0434	0.0520	0.0550	0.0261
0.0339	0.0415	0.0540	0.0582	0.0215
0.0396	0.0369	0.0480	0.0610	0.0251
0.0427	0.0395	0.0500	0.0616	0.0260
0.0378	0.0358	0.0510	0.0532	0.0240
0.0384	0.0476	0.0520	0.0540	0.0218
0.0374	0.0357	0.0581	0.0546	0.0174
0.0394	0.0344	0.0580	0.0545	0.0210
0.0402	0.0348	0.0583	0.0547	0.0184
0.0459	0.0399	0.0640	0.0638	0.0205
0.0483	0.0395	0.0653	0.0655	0.0184
0.0555	0.0396	0.0712	0.0721	0.0148
0.0601	0.0430	0.0799	0.0829	0.0056
0.0729	0.0489	0.0852	0.0903	0.0027
0.0608	0.0438	0.0775	0.0770	-0.0194
0.0612	0.0419	0.0764	0.0753	0.0226
0.0634	0.0451	0.0830	0.0819	0.0161
0.0505	0.0581	0.0922	0.0955	-0.0072
0.0739	0.0554	0.0910	0.0919	-0.0190
0.0735	0.0507	0.0899	0.0882	0.0190
0.0686	0.0555	0.0895	0.0868	0.0279
0.0738	0.0619	0.0968	0.0970	-0.0005
0.0847	0.0659	0.1115	0.1087	0.0116
0.1113	0.0691	0.1395	0.1344	0.0052
0.1259	0.0749	0.1652	0.1631	0.0158
0.1289	0.0794	0.1579	0.1531	-0.0120
0.1019	0.0648	0.1343	0.1311	0.0136

AETNA LIFE INSURANCE COMPANY

A. Holdings ($)

1.	2.	3.	4.	5.
01507784552	00149144799	00533927178	00025655881	02040556315
01646980612	00177864228	00597112433	00027668769	02243910654
01728602222	00196198626	00707924447	00030975777	02423732153
01802153853	00195796014	00847332026	00032613510	02617381117
01901419559	00185789173	00940232290	00043919981	02792743197
02036026932	00235265072	01033228778	00046214094	03027936023
02128764821	00255212009	01143691789	00048695719	03232088851
02208780981	00268250110	01274233128	00050451402	03443923936
02318149249	00299192296	01388391541	00053129126	03665484937
02468985377	00292077470	01482908444	00053074750	03888124908
02562178369	00343446512	01649812584	00066013222	04183614240
02628961990	00154871357	01880521616	00068458201	04368420703
02706681817	00155525084	02184936697	00066108356	04723670787
02790323467	00151242864	02444521935	00061337560	05023659022
02960656470	00187605744	02627227286	00063401678	05356850692
03069806211	00182892457	02783811212	00087927431	05424433808
03020803098	00131970614	02924165688	00089466096	05377275868
02957511365	00105787788	03108616448	00091110805	05410146076
03257990162	00109751509	03188518371	00110126011	05648604308
03665720994	00142486716	03272433394	00114971842	06681979267
03893923779	00116897294	03515037743	00130385771	06520262536
04067818308	00084297139	03845563379	00146958166	06381212506
04328894986	00237787770	04156584936	00173724446	07175639508
05604543551	00287902629	04430602729	00209339074	09339100886
06598238000	00135163000	04985112000	00261057000	10651347000
07325102000	00126071000	05940982000	00243112000	12002105000
07456689000	00242960000	06915860000	00236906000	11825416000
07689747000	00250845000	08316481000	00293984000	11342935000
06916485000	00253653000	09126167000	00303760000	08543998000
06916485000	00272835000	09528625000	00459836000	05319415000
06310612000	02517760000	09629406000	00547710000	03097284000

B. Expected Returns (percentage point)

1.	2.	3.	4.	5.
0.0282	0.0509	0.0500	0.0415	0.0487
0.0252	0.0450	0.0470	0.0435	0.0605
0.0271	0.0400	0.0490	0.0465	0.0477
0.0326	0.0408	0.0460	0.0518	0.0454
0.0352	0.0434	0.0520	0.0550	0.0416
0.0339	0.0415	0.0540	0.0582	0.0385
0.0396	0.0369	0.0480	0.0610	0.0306
0.0427	0.0395	0.0500	0.0616	0.0280
0.0378	0.0358	0.0510	0.0532	0.0303
0.0384	0.0476	0.0520	0.0540	0.0277
0.0374	0.0357	0.0581	0.0546	0.0279
0.0394	0.0344	0.0580	0.0545	0.0225
0.0402	0.0348	0.0583	0.0547	0.0233
0.0459	0.0399	0.0640	0.0638	0.0267
0.0483	0.0395	0.0653	0.0655	0.0234
0.0555	0.0396	0.0712	0.0721	0.0238
0.0601	0.0430	0.0799	0.0829	0.0226
0.0729	0.0489	0.0852	0.0903	0.0257
0.0608	0.0438	0.0775	0.0770	0.0314
0.0612	0.0419	0.0764	0.0753	0.0351
0.0634	0.0451	0.0830	0.0819	0.0301
0.0505	0.0581	0.0922	0.0955	0.0152
0.0739	0.0554	0.0910	0.0919	0.0141
0.0735	0.0507	0.0899	0.0882	0.0261
0.0686	0.0555	0.0895	0.0868	0.0359
0.0738	0.0619	0.0968	0.0970	0.0305
0.0847	0.0659	0.1115	0.1087	0.0319
0.1113	0.0691	0.1395	0.1344	0.0266
0.1259	0.0749	0.1652	0.1631	0.0369
0.1289	0.0794	0.1579	0.1531	0.0535
0.1019	0.0648	0.1343	0.1311	0.0476

NEW YORK LIFE INSURANCE COMPANY

A. Holdings ($)

1.	2.	3.	4.	5.
03323484459	00256881783	01424093378	00172184615	04758098460
03279480742	00398212223	00549417965	00191779360	04984444161
03310051837	00457285843	01674220088	00204473398	05194859941
03322716943	00434884234	01822913424	00218864108	05333517136
03351676742	00432654619	01904321370	00250017971	05459928346
03479675346	00499024661	01922857530	00279466296	05734329987
03553365271	00516153266	01939639880	00332591893	05883063095
03624132360	00562692512	01973515884	00333849727	05978385150
03692550525	00661101735	02029105061	00350002960	06206054244
03871605495	00648243309	02058502464	00367829155	06405622045
04015634458	00659705359	02191969215	00372177715	06699913133
04214868326	00685796367	02289144398	00366007966	06975038456
04388473132	00698796230	02451118050	00357199582	07278715082
04476142398	00634309410	02583274297	00351734321	07426840644
04653418566	00670589220	02659805501	00359834999	07722763721
04892736659	00712668304	02690241420	00361383387	08024849786
04940908969	00666524227	02777012279	00327755673	08088375923
04994846993	00678491781	02877270605	00321409818	08860569197
05267445999	00726076522	02953638918	00323175493	08684398728
05596863476	00840035121	03069852764	00273270937	09159149159
05774240485	00825081991	03247676533	00256838585	09479662880
05848676314	00706915986	03509622390	00285862792	09763606270
06202291899	00806092066	03673100389	00302116929	05549925171
06817319863	00843618545	03796277397	00321176446	06143054280
07484724000	00812679000	03923183000	00318915000	09236044000
08284982000	00816452000	04057318000	00347134000	09683766000
08708591000	00862760000	04476390000	00334956000	09699628000
08731787000	00898247000	05018801000	00332275000	10454562000
09007952000	00787410000	05349128000	00349531000	10466162000
09475996000	00865785000	05701398000	00414760000	10557504000
09839009000	00914194000	06083371000	00537171000	10679189000

B. Expected Returns (percentage point)

1.	2.	3.	4.	5.
0.0282	0.0509	0.0500	0.0415	0.0300
0.0252	0.0450	0.0470	0.0435	0.0332
0.0271	0.0400	0.0490	0.0465	0.0317
0.0326	0.0408	0.0460	0.0518	0.0269
0.0352	0.0434	0.0520	0.0550	0.0145
0.0339	0.0415	0.0540	0.0582	0.0302
0.0396	0.0369	0.0480	0.0610	0.0192
0.0427	0.0395	0.0500	0.0616	0.0158
0.0378	0.0358	0.0510	0.0532	0.0304
0.0384	0.0476	0.0520	0.0540	0.0211
0.0374	0.0357	0.0581	0.0546	0.0027
0.0394	0.0344	0.0580	0.0545	0.0082
0.0402	0.0348	0.0583	0.0547	-0.0015
0.0459	0.0399	0.0640	0.0638	0.0049
0.0483	0.0395	0.0653	0.0655	0.0062
0.0555	0.0396	0.0712	0.0721	0.0039
0.0601	0.0430	0.0799	0.0829	0.0019
0.0729	0.0489	0.0852	0.0903	0.0002
0.0608	0.0438	0.0775	0.0770	-0.0045
0.0612	0.0419	0.0764	0.0753	0.0068
0.0634	0.0451	0.0830	0.0819	0.0111
0.0505	0.0581	0.0922	0.0955	0.0117
0.0739	0.0554	0.0910	0.0919	0.0308
0.0735	0.0507	0.0899	0.0882	0.0174
0.0686	0.0555	0.0895	0.0868	0.0378
0.0738	0.0619	0.0968	0.0970	0.0386
0.0847	0.0659	0.1115	0.1087	0.0367
0.1113	0.0691	0.1395	0.1344	0.0188
0.1259	0.0749	0.1652	0.1631	0.0486
0.1289	0.0794	0.1579	0.1531	0.0254
0.1019	0.0648	0.1343	0.1311	0.0104

JOHN HANCOCK MUTUAL LIFE INSURANCE COMPANY

A. Holdings ($)

1.	2.	3.	4.	5.
02613303617	00189833918	00720791544	00068508378	03254748554
02768473117	00267967439	00267967439	00073918574	03584107070
02838381137	00315744493	01076746464	00076591384	03881327380
02980084524	00282039607	01252131188	00084985437	04126630139
03148343320	00249481377	01356425472	00091334449	04341724529
03372825092	00321781053	01395641499	00092789526	04671770853
03550974385	00350868399	01471305762	00101942016	04956601759
03703977269	00348169597	01564955948	00103301822	05184789089
03878094977	00411861709	01685851028	00104608745	05504170814
04070468895	00365362961	01794649293	00103765116	05764596944
04206364355	00390804644	01960242079	00120095035	06126883799
04054378644	00439833072	02228574788	00146545778	06390861771
04243851643	00466394622	02560213934	00165255114	06745708535
04233815258	00444327475	02805371119	00204032328	07029706061
04278228262	00556794081	02974571866	00252774493	07442335229
04238292234	00662038461	03170821335	00326462662	07802560091
04238172988	00579537685	03261725455	00430041877	07958067272
04298711640	00510973614	03398432465	00474966245	08140571782
04459528277	00632830184	03380561735	00488177243	08448315565
04684973316	00567773122	03757200083	00510583444	08967094529
04824545159	00488181533	04017231240	00483310194	09312215592
05236629977	00611604816	04206594469	00537988253	10053194290
05878584359	00705323162	04349398305	00694343854	11087236330
06445472000	00688647000	04653182000	00747343000	11997848000
07014137000	00744128000	05018137000	00728908000	12875892000
06871679000	00744128000	05018137000	00728908000	12733434000
07283308000	00761095000	05483223000	00715251000	13499713000
07209400000	00725700000	06372000000	00691600000	14198900000
06551500000	00964400000	06542000000	00797600000	13796100000
06390500000	00818500000	06381400000	00987100000	14042760000
06110100000	00837900000	06480600000	01133900000	13935800000

B. Expected Returns (percentage point)

1.	2.	3.	4.	5.
0.0282	0.0509	0.0500	0.0415	0.0452
0.0252	0.0450	0.0470	0.0435	0.0483
0.0271	0.0400	0.0490	0.0465	0.0621
0.0326	0.0408	0.0460	0.0518	0.0011
0.0352	0.0434	0.0520	0.0550	0.0481
0.0339	0.0415	0.0540	0.0582	0.0428
0.0396	0.0369	0.0480	0.0610	0.0423
0.0427	0.0395	0.0500	0.0616	0.0335
0.0378	0.0358	0.0510	0.0532	0.0452
0.0384	0.0476	0.0520	0.0540	0.0335
0.0374	0.0357	0.0581	0.0546	0.0308
0.0394	0.0344	0.0580	0.0545	0.0264
0.0402	0.0348	0.0583	0.0547	0.0315
0.0459	0.0399	0.0640	0.0638	0.0184
0.0483	0.0395	0.0653	0.0655	0.0141
0.0555	0.0396	0.0712	0.0721	0.0303
0.0601	0.0430	0.0799	0.0829	0.0084
0.0729	0.0489	0.0852	0.0903	0.0101
0.0608	0.0438	0.0775	0.0770	0.0117
0.0612	0.0419	0.0764	0.0753	0.0196
0.0634	0.0451	0.0830	0.0819	0.0239
0.0505	0.0581	0.0922	0.0955	0.0071
0.0739	0.0554	0.0910	0.0919	0.0150
0.0735	0.0507	0.0899	0.0882	0.0196
0.0686	0.0555	0.0895	0.0868	0.0367
0.0738	0.0619	0.0968	0.0970	0.0345
0.0847	0.0659	0.1115	0.1087	0.0317
0.1113	0.0691	0.1395	0.1344	0.0369
0.1259	0.0749	0.1652	0.1631	0.0071
0.1289	0.0794	0.1579	0.1531	0.0335
0.1019	0.0648	0.1343	0.1311	0.0218

CONNECTICUT GENERAL LIFE INSURANCE COMPANY

A. Holdings ($)

1.	2.	3.	4.	5.
00640219662	00023107767	00420422148	00031487373	01160960359
00706210614	00026879696	00465080688	00037974070	01088178829
00750484315	00030923932	00553216084	00050285881	01276964741
00809353382	00031845807	00613536627	00063770898	01400383441
00890247145	00032610204	00665586009	00069432053	01530779733
00998050189	00041150034	00710019175	00072210782	01696579259
01092954688	00047362440	00757871311	00077430230	01841121290
01173830227	00053720483	00804057377	00076977958	01964908295
01251789574	00073441696	00878071809	00076408441	02122821166
01347484970	00073322641	00945735710	00077912524	02279921752
01465976260	00079561077	01030064432	00082538583	02484644677
01548250213	00094487218	01141731568	00082124153	02670072876
01545654640	00104276153	01298422274	00095279503	02869908724
01731850844	00098700460	01412685683	00113142040	03049688910
01874728940	00117541833	01542011043	00135079730	03419105725
01945980223	00136176223	01685033891	00154915099	03243737748
01983542971	00161281717	01801688511	00163452820	03569201643
02091584815	00123857032	01896866435	00193140727	03720642800
02417260437	00151328908	01951351503	00215223254	03989055159
02673967596	00178164768	02108921628	00219015070	04319132809
02850008277	00137401888	02321114842	00228713780	04661592707
03016716921	00097743804	02507684984	00239522231	05050729999
03321915630	03708882527	02671251572	00248078089	05396664917
03321915630	03018829445	02961247981	00252233774	06027193530
04343863000	00211345000	03372313000	00258948000	06820283000
04943600000	00549000000	03857704000	00221489000	07480532000
05516857000	00048601000	04326452000	00200679000	08481092000
05871723000	00063985000	04765567000	00217369000	08743707000
05833054000	00048076000	05144568000	00217599000	08743957000
05431218000	00052269000	05339319000	00372676000	08969765000
05761876000	00055259000	05975440000	00408812000	08207406000

B. Expected Returns (percentage point)

1.	2.	3.	4.	5.
0.0282	0.0509	0.0500	0.0415	0.0465
0.0252	0.0450	0.0470	0.0435	0.0402
0.0271	0.0400	0.0490	0.0465	0.0362
0.0326	0.0408	0.0460	0.0518	0.0359
0.0352	0.0434	0.0520	0.0550	0.0290
0.0339	0.0415	0.0540	0.0582	0.0309
0.0396	0.0369	0.0480	0.0610	0.0323
0.0427	0.0395	0.0500	0.0616	0.0326
0.0378	0.0358	0.0510	0.0532	0.0319
0.0384	0.0476	0.0520	0.0540	0.0328
0.0374	0.0357	0.0581	0.0546	0.0297
0.0394	0.0344	0.0580	0.0545	0.0310
0.0402	0.0348	0.0583	0.0547	0.0301
0.0459	0.0399	0.0640	0.0638	0.0318
0.0483	0.0395	0.0653	0.0655	0.0336
0.0555	0.0396	0.0712	0.0721	0.0367
0.0601	0.0430	0.0799	0.0829	0.0320
0.0729	0.0489	0.0852	0.0903	0.0102
0.0608	0.0438	0.0775	0.0770	0.0293
0.0612	0.0419	0.0764	0.0753	0.0527
0.0634	0.0451	0.0830	0.0819	0.0540
0.0505	0.0581	0.0922	0.0955	0.0535
0.0739	0.0554	0.0910	0.0919	0.0473
0.0735	0.0507	0.0899	0.0882	0.0405
0.0686	0.0555	0.0895	0.0868	0.0398
0.0738	0.0619	0.0968	0.0970	0.0365
0.0847	0.0659	0.1115	0.1087	0.0414
0.1113	0.0691	0.1395	0.1344	0.0499
0.1259	0.0749	0.1652	0.1631	0.0440
0.1289	0.0794	0.1579	0.1531	0.0620
0.1019	0.0648	0.1343	0.1311	0.0590

A. Holdings ($)

1.	2.	3.	4.	5.
01049825213	00093882736	00377378202	00043638819	01468200835
01079956709	00125911390	00420445533	00048649618	01573196789
01096966439	00145256614	00483036871	00054218618	01671865381
01108264606	00138513180	00571391648	00055639489	01748078244
01129958482	00117174100	00643688268	00052626270	01816161671
01153497428	00150116113	00703024920	00061677505	01932972330
01194742453	00173097958	00729599286	00062732324	02020388181
01231593789	00183278333	00751446931	00067231613	02094249577
01308774125	00207836136	00775738769	00074551477	02213228473
01385382242	00214689513	00796826886	00075115010	02315399621
01431218801	00231840953	00855394203	00077450826	02436176275
01443084396	00265522755	00955055183	00085402164	02581285893
01519063839	00291525796	01009914682	00086337457	02703223075
01590756370	00228148150	01054269761	00079290169	02741389479
01657388597	00244890616	01122616150	00093229548	02873716981
01739837021	00263305956	01166688551	00108298958	02995162851
01721478962	00196865313	01200666437	00125090732	02951270142
01733682251	00182580446	01268042672	00148129154	03009686009
01837860514	00223377036	01295382417	00151726176	03084445152
01943123515	00290714459	01354676983	00155515515	03339234947
01951285233	00258132738	01463526020	00183375459	03327366369
01989364152	00176960834	01637662396	00166735189	03308889523
02048243359	00174224293	01892232365	00164492965	03558647409
02339968924	00212513716	01953382333	00180051524	03851565873
02700784000	00209072000	01993052000	00212188000	04218622000
02982520000	00232395000	02170586000	00214604000	03134401000
03092542000	00266407000	02436863000	00221966000	04882978000
03279664000	00223752000	02689301000	00186322000	05031028000
03017612000	00223629000	02917669000	00198791000	05304216000
03527255000	00273352000	02897092000	00214452000	05322979000
04399077000	00292818000	02773931000	00179795000	05792942000

B. Expected Returns (percentage point)

1.	2.	3.	4.	5.
0.0282	0.0509	0.0500	0.0415	0.0394
0.0252	0.0450	0.0470	0.0435	0.0414
0.0271	0.0400	0.0490	0.0465	0.0379
0.0326	0.0408	0.0460	0.0518	0.0334
0.0352	0.0434	0.0520	0.0550	0.0155
0.0339	0.0415	0.0540	0.0582	0.0037
0.0396	0.0369	0.0480	0.0610	0.0063
0.0427	0.0395	0.0500	0.0616	0.0102
0.0378	0.0358	0.0510	0.0532	0.0273
0.0384	0.0476	0.0520	0.0540	0.0146
0.0374	0.0357	0.0581	0.0546	0.0164
0.0394	0.0344	0.0580	0.0545	0.0192
0.0402	0.0348	0.0583	0.0547	0.0269
0.0459	0.0399	0.0640	0.0638	0.0260
0.0483	0.0395	0.0653	0.0655	0.0166
0.0555	0.0396	0.0712	0.0721	0.0081
0.0601	0.0430	0.0799	0.0829	0.0431
0.0729	0.0489	0.0852	0.0903	0.0127
0.0608	0.0438	0.0775	0.0770	0.0127
0.0612	0.0419	0.0764	0.0753	0.0165
0.0634	0.0451	0.0830	0.0819	0.0223
0.0505	0.0581	0.0922	0.0955	0.0182
0.0739	0.0554	0.0910	0.0919	0.0190
0.0735	0.0507	0.0899	0.0882	0.0220
0.0686	0.0555	0.0895	0.0868	0.0237
0.0738	0.0619	0.0968	0.0970	0.0201
0.0847	0.0659	0.1115	0.1087	0.0304
0.1113	0.0691	0.1395	0.1344	0.0330
0.1259	0.0749	0.1652	0.1631	0.0409
0.1289	0.0794	0.1579	0.1531	0.0567
0.1019	0.0648	0.1343	0.1311	0.0329

DEFINITIONS :

1 - BONDS
2 - STOCKS
3 - MORTGAGE
4 - REALESTATE

SOURCE: Moody's Bank & Finance Manual, various years
Statistical Abstract of the United States, various years

CHAPTER 6

Conclusions

The statistical outline in Chapter 1 provided evidence of the growing importance of insurance companies in the United States. Given that life insurance companies serve as a major supplier of funds to the capital market, any change in their product mix for investment will have a significant effect on the movement of the market, that is the relative rates of return on specific instruments. Therefore, estimating the own and cross elasticities does have certain important economic policy implications for the financial assets (liability) held by the life insurance companies.

The methodology for this study has been based on a synthesis of portfolio theory and the use of flexible functional forms in demand-system analysis. This latter function takes on the translog, the generalized Leontief, the square root quadratic and quadratic utility function as special or limiting cases. Budget-share equations for assets (liability) are derived from the generalized Box-Cox utility function through the use of the Seemingly Unrelated Regression (SUR) estimation technique. Utilizing the theory of asset demand and a Chi-square test based on the estimated budget-share equation, it was determined that the quadratic appears to describe best the preferences for the life insurance companies in the United States. The empirical results for the quadratic utility function indicated that for six out of eight life insurance companies, the own elasticities of demand with respect to expected return, and the own elasticities of demand with respect to the variance (risk), had signs predicted by theory. The estimates of the cross elasticities among the assets (liability) revealed a relationship of substitution between real estate and stocks, and one of complementary between bonds and stocks. The overall results also showed that stocks are the most risky assets compared to bonds, mortgage or real estate.

One of the limitations of the study is that bonds and stocks were classified as a single group. Ideally, one would desire to divide the bonds and stocks into different classifications, and also to have more than one liability measure in order to have a more specific analysis of the nature of the life insurers' investment behaviour. However, a detailed breakdown of such information was not available.

In addition, the expected returns were specified and calculated based on the simple adaptive expectation scheme, whereas other approaches such as rational expectations are more general. However, given the complexity of the estimated model, the introduction of the rational expectations is beyond the scope of the present study.

Finally, nominal interest rates are used in the study. Thus it was assumed that all financial assets and liabilities are equally affected by inflation. Since inflation is an important phenomenon in modern society, a more explicit treatment could be considered desirable. A suggestion for further research is the use of expected real interest rates, based on expected inflation.

The study of the investment attitudes by the mutual life and the stock life companies has also indicated that the mutual life companies appear to adopt a more aggressive investment strategy as compared to the stock life companies. This result could explain why the mutual life companies have been able to accumulate 70% of the total assets, and thus dominate the life insurance industry in the United States.

Given no evidence as yet of a slackening in the pace of expansion in the life insurance industry, there is every likelihood that the insurance industry will continue to control a large proportion of personal savings for many years to come. As a result, the life insurance companies are expected to have a continuing significant influence on financial markets.

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